International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 2 Issue 11 ǁ November. 2014 ǁ PP.01-08

M^x/G(a,b)/1 With Modified Vacation, Variant Arrival Rate With Restricted Admissability Of Arriving Batches And Close Down

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ABSTRACT: *In this paper, a bulk arrival general bulk service queuing system with modified M-vacation policy, variant arrival rate under a restricted admissibility policy of arriving batches and close down time is considered. During the server is in non- vacation, the arrivals are admitted with probability with ' α ' whereas, with probability 'β' they are admitted when the server is in vacation. The server starts the service only if at least 'a' customers are waiting in the queue, and renders the service according to the general bulk service rule with minimum of 'a' customers and maximum of 'b' customers. At the completion of service, if the number of waiting customers in the queue is less than 'a' then the server performs closedown work , then the server will avail of multiple vacations till the queue length reaches a consecutively avail of M number of vacations, After completing the Mth vacation, if the queue length is still less than a then the server remains idle till it reaches a. The server starts the service only if the queue length b $\geq$ a. It is considered that the variant arrival rate dependent on the state of the server.*

keywords: *General Bulk service, multiple vacation, restricted admissibility and close down, M- vacation.*

I. INTRODUCTION

Server vacation models are useful for the systems in which the server wants to utilize the idle time for different purposes. Application of server vacation models can be found in manufacturing systems, designing of local area networks and data communication systems. Very few authours are works on queueing systems with closedown time. An M/G/1 queue is analyzed by Takagi considered closedown time and set up time .It is observed that most of the studies on vacation queue are concentrated only on single server on single arrival and single vacation. Once the arrival occur in bulk one expect that the server can also be done in bulk.

Li and Zhu (1997) investigated a single arrival, single service finite queue with generalised vacations and exhaustive service where arrival rates depend on the number of customers in the system. Batch arrival queueing systems with vacations were developed by several researches such as Lee.et. al (1994), Ke and Chang (2009) etc. Parathasarathy and Sudesh (2010) have discussed a state- dependent queue alternating between arrivals and services, in which they obtained time-dependent system size probablities and the duration of the busy period in a close form. Wang et.al (2007) have analyzed a single unreliable server in an M^x/M/1 queueing system with multiple vacations, Ke (2007) discussed the operating characteristics of an M^xG/1 queueing system under vacation policies with startup/ closedown times are generally distributed. Choudhury and Madan (2007) have considered a batch arrival but single service Bernoulli vacation queue, with a random setup time under restricted admissiblity policy. Sikdar and Gupta (2008) have discussed on the batch arrival, batch service queue, but finite buffer under server's vacation. M^x/M^y /1/N queue. Jain and Upadhyaya (2010) considered with the modified Bernoulli vacation schedule for the unreliable server batch arrival queueing system with essential and milti-optional services under N- policy.

II. Notations

Let X be the group size random variable of the arrival λ, be the Poisson arrival rate.$g_k$be the probability that 'k' customers arrive in a batch and X (z) be its probability generating function (PGF).

S(.) Cumulative distribution function of service time.
V(.) Cumulative distribution function of vacation time.
C(.) Cumulative distribution function of closedown time.
s(x) Probability density function of S and$\tilde{S}(\theta)$ be the Laplace-Stieltjes transform of S
v(x) Probability density function of V and$\tilde{V}(\theta)$ be the Laplace-Stieltjes transform of V
c(x) Probability density function of C and $\tilde{C}(\theta)$ be the Laplace-Stieltjes transform of C
$S^0(t)$Remaining service time of a batch in service at time 't'
$V^0(t)$Remaining vacation time at time 't'
$C^0(t)$Remaining closedown time at time 't'
$N_s(t)$= Number of customers in the service at time t
$N_q(t)$= Number of customers in the queue at time t

The different states of the server at time t are defined as follows

C(t) = 0 ; if the server is busy with service

1 ; if the server is on vacation

2; if the server is on dormant period

3; if the server is on closedown time

z(t) = j , if the server is on j th vacation starting from the idle period.

To obtain the system equations , the following state probabilities are defined;

$* P_{i,n}(x,t)dt = P\{N_s(t) = i, N_q(t) = n, x \leq S^0(t) \leq x + dt, C(t) = 0\}, a \leq i \leq b, n \geq 0,$

is the joint probability that at time t, the server is busy and rendering service , the queue size is j, the number of customers under the service is i and the remaining service time of a batch is x.

$* Q_n(x,t)dt = P\{ N_q(t) = n, x \leq V^0(t) \leq x + dt, C(t) = 1\}, n \geq 0$

is the is the joint probability that at time t, the server is on the jth vacation starting from the idle period and the remaining vacation time is x .

$$* T_n(t)dt = P\{ N_q(t) = n, C(t) = 2\}, 0 \leq n \leq a - 1.$$

is the probability at time t, the queue size is n and the server is on dormant period.

$* C_n(x,t)dt = P\{ N_q(t) = n, x \leq C^0(t) \leq x + dt, C(t) = 3\}, n \geq 0$

is the probability at time t, the queue size is n and the server is on closedown period

Now, the following system equations are obtained for the queueing system, using supplementary variable technique:

$$T_0(t + \Delta t) = T_0(t)(1 - \lambda_0 \Delta t) + Q_{M,0}(0,t)\Delta t$$

$$T_n(t + \Delta t) = T_n(t)(1 - \lambda_0 \Delta t) + Q_{M,n}(0,t)\Delta t + \sum_{k=1}^{n} T_{n-k}(t)\lambda_0 g_k \, \Delta t, \qquad 1 \leq n \leq a - 1,$$

$$P_{i,0}(x - \Delta t, t + \Delta t) = P_{i,0}(x,t)(1 - \lambda \Delta t) + \lambda(1 - \alpha)P_{i,0}(0,t) + \sum_{m=a}^{b} P_{m,i}(0,t)s(x)\Delta t + R_i(0)s(x)$$
$$+ \sum_{k=1}^{M} Q_{kI}(0,t)\lambda g_{i-k} \, s(x)\Delta t + \sum_{m=0}^{a-1} T_m(t)\lambda g_{i-m} \, s(x)\Delta t, ; \; a \leq i \leq b$$

$$P_{i,j}(x - \Delta t, t + \Delta t) = P_{i,j}(x,t)(1 - \lambda \Delta t) + \lambda(1 - \alpha)P_{i,j}(x,t) + \alpha \sum_{k=1}^{j} P_{i,j-k}(x,t)\lambda g_k \, \Delta t \, ;$$
$$a \leq i \leq b - 1; j \geq 1$$

$$P_{b,j}(x - \Delta t, t + \Delta t) = P_{b,j}(x,t)(1 - \lambda \Delta t) + \lambda(1 - \alpha)P_{b,j}(x,t) + \sum_{m=a}^{b} P_{m,b+j}(0,t)s(x)\Delta t +$$
$$+\alpha \sum_{k=1}^{j} P_{b,j-k}(x,t)\lambda g_k \, \Delta t + \sum_{m=0}^{a-1} T_m(t)\lambda g_{b+j-m} \, s(x)\Delta t + \sum_{j=0}^{M} Q_{l,b+j}(0,t)\lambda g_{b+j-k} \, s(x)\Delta t, j \geq 1$$

$$C_n(x - \Delta t, t + \Delta t) = C_n(x,t)(1 - \lambda_0 \Delta t) + \lambda_0(1 - \alpha)C_n(x,t)\Delta t + \sum_{m=a}^{b} P_{m,n}(0,t)c(x)\Delta t +$$
$$+\alpha \textstyle\sum_{k=1}^{j} C_{n-k}(x,t)\lambda_0 g_k \, \Delta t, \quad n \leq a-1$$

$$C_n(x - \Delta t, t + \Delta t) = C_n(x,t)(1 - \lambda \Delta t) + \lambda_0(1 - \alpha)C_n(x,t)\Delta t + +\alpha \textstyle\sum_{k=1}^{j} C_{n-k}(x,t)\lambda_0 g_k \, \Delta t, \quad n \geq a$$

$$Q_{l,0}(x - \Delta t, t + \Delta t) = Q_{l,0}(x,t)(1 - \lambda_0 \Delta t) + \lambda_0(1 - \beta)Q_{l,0}(x,t)\Delta t + C_0(0,t) \, v \, (x) \, \Delta t$$

$$Q_{l,n}(x - \Delta t, t + \Delta t) = Q_{l,n}(x,t)(1 - \lambda_0 \Delta t) + \lambda_0(1 - \beta)Q_{l,n}(x,t)\Delta t + \sum_{m=a}^{b} P_{m,n}(0,t)v(x)\Delta t$$
$$+ \sum_{k=1}^{n} Q_{l,n-k}(x,t)\lambda_0 g_k \, \Delta t + C_n(0,t)v(x)\Delta t \qquad 1 \leq n \leq a - 1$$

$$Q_{l,n}(x-\Delta t,t+\Delta t)$$
$$=Q_{l,n}(x,t)(1-\lambda_0\Delta t)+\lambda_0(1-\beta)Q_{l,n}(x,t)\Delta t+\sum_{k=1}^{n}Q_{l,n-k}(x,t)\lambda_0 g_k\,\Delta t+C_n(0,t)v(x)\Delta t;\quad n\geq a$$

$$Q_{j,0}(x-\Delta t,t+\Delta t)=Q_{j,0}(x,t)(1-\lambda_0\Delta t)+\lambda_0(1-\beta)Q_{j,0}(x,t)\Delta t+Q_{j-1,0}(0,t)v(x)\Delta t\quad 2\leq j\leq M$$

$$Q_{j,n}(x-\Delta t,t+\Delta t)=Q_{j,n}(x,t)(1-\lambda_0\Delta t)+\lambda_0(1-\beta)Q_{j,n}(x,t)\Delta t+\sum_{k=1}^{n}Q_{j,n-k}(x,t)\lambda_0 g_k\,\Delta t;$$
$$+Q_{j-1,n}(0,t)v(x)\qquad ,1\leq n\leq a-1,2\leq j\leq M$$

$$Q_{j,n}(x-\Delta t,t+\Delta t)=Q_{j,n}(x,t)(1-\lambda_0\Delta t)+\lambda_0(1-\beta)Q_{j,n}(x,t)\Delta t+\sum_{k=1}^{n}Q_{j,n-k}(x,t)\lambda_0 g_k\,\Delta t\,;\quad n\geq a,2\leq j\leq M$$

III. STEADY STATE QUEUE SIZE DISTRIBUTION

From the above equations, the steady state queue size equations are obtained as follows:

$$0=-\lambda_0T_0+Q_{M,0}(0)\tag{1}$$

$$0=-\lambda_0T_n+Q_{M,n}(0)+\sum_{k=1}^{n}T_{n-k}\lambda_0\,g_k\quad ,1\leq n\leq a-1\tag{2}$$

$$-P_{i,0}^{'}(x)=-\lambda P_{i,0}(x)+\lambda(1-\alpha)P_{i,0}(x)+\sum_{m=a}^{b}P_{m,i}(0)s(x)\Delta t$$
$$+\sum_{l=1}^{M}Q_k(0)s(x)+\lambda\sum_{m=0}^{a-1}T_m\lambda g_{i-m}\,s(x)\quad a\leq i\leq b\tag{3}$$

$$-P_{i,j}^{'}(x)=-\lambda P_{i,j}(x)+\lambda(1-\alpha)P_{i,j}(x)+\alpha\sum_{k=1}^{j}P_{i,j-k}(x)\,\lambda g_k\,;\qquad a\leq i\leq b\,,\mathrm{j}\geq 1\tag{4}$$

$$-P_{b,j}^{'}(x)=-\lambda P_{b,j}(x)+\lambda(1-\alpha)P_{b,j}(x)+\alpha\sum_{k=1}^{j}P_{b,j-k}(x)\,\lambda g_k+\sum_{m=a}^{b}P_{m,b+j}(0)s(x)$$
$$+\sum_{m=0}^{n-1}T_m\lambda g_{b+j-m}\,s(x)+\sum_{i=1}^{M}Q_{l,b+i}(0)s(x)+R_{b+j}(0)s(x);\quad j\geq 1\tag{5}$$

$$-C_n^{'}(x)=-\lambda_0C_n(x)+\lambda_0(1-\alpha)C_n(x)+\alpha\textstyle\sum_{k=1}^{n}C_{n-k}(x)\,\lambda_0 g_k+\sum_{m=a}^{b}P_{m,n}(0)s(x),\quad n\leq a-1\tag{6}$$

$$-C_n^{'}(x)=-\lambda_0C_n(x)+\lambda_0(1-\alpha)C_n(x)+\alpha\textstyle\sum_{k=1}^{n}C_{n-k}(x)\,\lambda_0 g_k\,,\quad n\geq a\tag{7}$$

$$-Q_{l,0}^{'}(x)=-\lambda_0Q_{l,0}(x)+\lambda_0(1-\beta)Q_{l,0}(x)+C_0(0)v(x);\tag{8}$$

$$-Q_{l,n}^{'}(x)=-\lambda_0Q_{l,n}(x)+\lambda_0(1-\beta)Q_{l,n}(x)+\beta\sum_{k=1}^{n}Q_{l,n-k}(x)\lambda_0 g_k+C_n(0)v(x);$$
$$1\leq n\leq a-1\tag{9}$$

$$-Q_{l,n}^{'}(x)=-\lambda_0Q_{l,n}(x)+\lambda_0(1-\beta)Q_{l,n}(x)+\beta\textstyle\sum_{k=1}^{n}Q_{l,n-k}(x)\lambda_0 g_k+C_n(0)v(x);\ n\geq a\tag{10}$$

$$-Q_{j,0}'(x) = -\lambda_0 Q_{j,0}(x) + \lambda_0(1-\beta)Q_{j,0}(x) + Q_{j-1,0}(x)v(x);\ \ 2 \le j \le M \tag{11}$$

$$-Q_{j,n}'(x) = -\lambda_0 Q_{j,n}(x) + \lambda_0(1-\beta)Q_{j,n}(x) + Q_{j-1,0}(x)v(x) + \beta\sum_{k=1}^{n} Q_{j,n-k}(x)\lambda_0 g_k;\quad 1 \le n \le a-1,\quad 2 \le j \le M \tag{12}$$

$$-Q_{j,n}'(x) = -\lambda_0 Q_{j,n}(x) + \lambda_0(1-\beta)Q_{j,n}(x) + \beta\sum_{k=1}^{n} Q_{j,n-k}(x)\lambda_0 g_k;\ \ n \ge a, 2 \le j \le M \tag{13}$$

The Laplace-Stieltjes transforms of $P_{i,n}(x)$ and $Q_j(x)$ are defined as:
$\tilde{P}_{i,n}(\theta) = \int_0^\infty e^{-\theta x} P_{i,n}(x)dx$; $\tilde{Q}_j(\theta) = \int_0^\infty e^{-\theta x} Q_j(x)dx$ and $\tilde{C}_n(\theta) = \int_0^\infty e^{-\theta x} C_n(x)dx$
Taking Laplace-Stieltjes transform on both sides, we get

$$\theta\tilde{P}_{i,0}(\theta) - P_{i,0}(0) = \lambda\tilde{P}_{i,0}(\theta) - \lambda(1-\alpha)\tilde{P}_{i,0}(\theta) - \left[\sum_{m=a}^{b} P_{m,i}(0) + \sum_{l=1}^{M} Q_{l,i}(0)\lambda g_{i-k} + \sum_{m=0}^{a-1} T_m \lambda g_{i-m}\right]\tilde{S}(\theta);\quad a \le i \le b, \tag{14}$$

$$\theta\tilde{P}_{i,j}(\theta) - P_{i,j}(0) = \lambda\tilde{P}_{i,j}(\theta) - \lambda(1-\alpha)\tilde{P}_{i,j}(\theta) - \lambda\alpha\sum_{k=1}^{j}\tilde{P}_{i,j-k}(\theta)\, g_k,\quad a \le i < b-1, j \ge 1 \tag{15}$$

$$\theta\tilde{P}_{b,j}(\theta) - P_{b,j}(0) = \lambda\tilde{P}_{b,j}(\theta) - \lambda(1-\alpha)\tilde{P}_{b,j}(\theta) - \alpha\sum_{k=1}^{j}\tilde{P}_{b,j-k}(\theta)\,\lambda g_k - \sum_{m=a}^{b} P_{m,b+j}(0)\tilde{S}(\theta) - \sum_{m=0}^{n-1} T_m\, \lambda g_{b+j-m}\,\tilde{S}(\theta) - \sum_{l=1}^{M}\tilde{Q}_{l,b+j}(\theta)\tilde{S}(\theta),\quad j \ge 1 \tag{16}$$

$$\theta\tilde{C}_n(\theta) - C_n(0) = \lambda_0\tilde{C}_n(\theta) - \lambda_0(1-\beta)\tilde{C}_n(\theta) - \sum_{m=a}^{b} P_{m,0}(0)\tilde{S}(\theta) - \alpha\sum_{k=1}^{n}\tilde{C}_{n-k}(\theta)\,\lambda_0 g_k,\quad n \le a-1 \tag{17}$$

$$\theta\tilde{C}_n(\theta) - C_n(0) = \lambda_0\tilde{C}_n(\theta) - \lambda_0(1-\beta)\tilde{C}_n(\theta) - \alpha\sum_{k=1}^{n}\tilde{C}_{n-k}(\theta)\,(17) g_k,\ n \ge a \tag{18}$$

$$\theta\tilde{Q}_{l,0}(\theta) - Q_{l,0}(0) = \lambda_0\tilde{Q}_{l,0}(\theta) - \lambda_0(1-\beta)\tilde{Q}_{l,0}(\theta) - C_0(0)\tilde{V}(\theta); \tag{19}$$

$$\theta\tilde{Q}_{l,n}(\theta) - Q_{l,n}(0) = \lambda_0\tilde{Q}_{l,n}(\theta) - \lambda_0(1-\beta)\tilde{Q}_{l,n}(\theta) - \lambda_0\beta\sum_{k=1}^{j}\tilde{Q}_{l,n-k}(\theta)\, g_k - C_n(0)\tilde{V}(\theta);\quad 1 \le n \le a-1 \tag{20}$$

$$\theta\tilde{Q}_{l,n}(\theta) - Q_{l,n}(0) = \lambda_0\tilde{Q}_{l,n}(\theta) - \lambda_0(1-\beta)\tilde{Q}_{l,n}(\theta) - \lambda_0\beta\sum_{k=1}^{j}\tilde{Q}_{l,n-k}(\theta)\, g_k - C_n(0)\tilde{V}(\theta),\ n \ge a \tag{21}$$

$$\theta\tilde{Q}_{j,0}(\theta) - Q_{j,0}(0) = \lambda_0\tilde{Q}_{j,0}(\theta) - \lambda_0(1-\beta)\tilde{Q}_{j,0}(\theta) - \tilde{Q}_{j-1,0}(\theta)\tilde{V}(\theta);\qquad 2 \le j \le M \tag{22}$$

$$\theta\tilde{Q}_{j,n}(\theta) - Q_{j,n}(0) = \lambda_0\tilde{Q}_{j,n}(\theta) - \lambda_0(1-\beta)\tilde{Q}_{j,n}(\theta) - \lambda_0\beta\sum_{k=1}^{j}\tilde{Q}_{j,n-k}(\theta)\, g_k - Q_{j-1,n}(0)\tilde{V}(\theta);\quad 2 \le j \le M,\ 1 \le n \le a-1 \tag{23}$$

$$\theta\tilde{Q}_{j,n}(\theta) - Q_{j,n}(0) = \lambda_0\tilde{Q}_{j,n}(\theta) - \lambda_0(1-\beta)\tilde{Q}_{j,n}(\theta) - \lambda_0\beta\sum_{k=1}^{j}\tilde{Q}_{l,n-k}(\theta)\, g_k;\ \ 2 \le j \le M, n \ge a \tag{24}$$

IV. SYSTEM SIZE DISTRIBUTION

To obtain the system size distribution let us define PGF's as follows:

$$\tilde{P}_i(z,\theta) = \sum_{n=0}^{\infty}\tilde{P}_{i,n}(\theta)z^n\quad;\quad P_i(z,0) = \sum_{n=0}^{\infty}P_{i,n}(0)z^n; a \le i \le b,$$
$$\tilde{Q}_j(z,\theta) = \sum_{n=0}^{\infty}\tilde{Q}_{j,n}(\theta)z^n\quad;\quad Q_j(z,0) = \sum_{j=0}^{a-1}Q_{j,n}(0)z^n$$
$$\tilde{C}(z,\theta) = \sum_{n=0}^{\infty}\tilde{C}_n(\theta)z^n\quad;\quad C(z,0) = \sum_{n=0}^{\infty}C_n(0)z^n;$$
$$T(z) = \sum_{j=0}^{a-1}T_n z^n \tag{25}$$

The probability generating function P(z) of the number of customers in the queue at an arbitrary time epoch of the proposed model can be obtained using the following equation

$$P(z) = \sum_{i=a}^{b-1}\tilde{P}_i(z,0) + \tilde{P}_b(z,0) + \sum_{j=1}^{M}\tilde{Q}_j(z,0) + \tilde{C}(z,0) + T(z) \tag{26}$$

In order to find the following $\tilde{P}_i(z,\theta), \tilde{P}_b(z,\theta)$, $\tilde{Q}(z,\theta)$ and $\tilde{C}(z,\theta)$ sequence of operations are done.

Multiply the equations (19) by z^0, (20) by z^n $(1 < n < a-1)$ and (21) by z^n $(n \geq a)$, summing up from $n = 0$ to ∞ and by using (25), we get

$$\left[\theta - \beta\left(\lambda_0 - \lambda_0 X(z)\right)\right]\tilde{Q}_l(z,\theta) = Q_l(z,0) - C(z,0)\tilde{V}(\theta) \tag{27}$$

Multiply the equations (22) by z^0, (23) by z^n $(1 < n < a-1)$ and (24) by z^n $(n \geq a)$, summing up from $n = 0$ to ∞ and by using (25), we get

$$\left[\theta - \beta\left(\lambda_0 - \lambda_0 X(z)\right)\right]\tilde{Q}_j(z,\theta) = Q_j(z,0) - \tilde{V}(\theta)\sum_{n=0}^{a-1} Q_{j-1,n}(0)\, z^n\,;\ 2 \leq \mathrm{j} \leq \mathrm{M} \tag{28}$$

Multiply the equations (17) by $z^0 1 < n < a-1$)and(18)by z^n $(n > a)$ and summing up from $n = 0$ $to\infty$ and by using (25), we get

$$\left[\theta - \alpha\left(\lambda_0 - \lambda_0 X(z)\right)\right]\tilde{C}(z,\theta) = C(z,0) - \tilde{C}(\theta)\sum_{n=0}^{a-1}\left[\sum_{m=a}^{b} P_{m,n}(0)\right] z^n \tag{29}$$

Multiply the equations (14) by z^0, (15) by z^j $(j > a)$ and summing up from $n = 0$ to ∞ and using (25), we get

$$\left[\theta - \alpha\left(\lambda - \lambda X(z)\right)\right]\tilde{P}_i(z,\theta) = P_i(z,0) - \tilde{S}(\theta)\left[\sum_{m=a}^{b} P_{m,i}(0) + \sum_{l=1}^{M} Q_{l,i}(0)\lambda g_{i-k} + \sum_{m=0}^{n-1} T_m \lambda g_{i-m}\right]$$
$$a \leq i \leq b-1, \tag{30}$$

Multiply the equations (17) byz^0 (18) z^j $(j > a)$ and summing up from $j = 0$ to ∞ and using (27), we get

$$z^b\left[\theta - \alpha\left(\lambda - \lambda X(z)\right)\right]\tilde{P}_b(z,\theta) = z^b\ P_b(z,0)\text{-}\tilde{S}(\theta)\begin{bmatrix} \sum_{m=a}^{b-1} P_m(z,0) - \sum_{m=a}^{b}\sum_{j=0}^{b-1} P_{m,j}(0) z^j \\ +\lambda\left(T(z)X(z) - \sum_{m=0}^{a-1}\left(T_m z^m \sum_{\mathrm{j}=1}^{\mathrm{b-m-1}} \mathrm{g_j z^j}\right)\right) \\ \sum_{l=1}^{M} Q_l(z,0) - \sum_{l=1}^{M}\sum_{j=0}^{b-1} Q_{l,j}(0) z^j \end{bmatrix} \tag{31}$$

By substituting $\theta = \beta\left(\lambda - \lambda X(z)\right)$ in the equations (27), (28) we get

$$Q_l(z,0) = \tilde{V}\left(\beta\left(\lambda_0 - \lambda_0 X(z)\right)\right) C(z,0) \tag{32}$$

$$Q_j(z,0) = \tilde{V}\left(\beta\left(\lambda_0 - \lambda_0 X(z)\right)\right)\sum_{n=0}^{a-1} Q_{j-1,n}(0)\, z^n\,;\ 2 \leq \mathrm{j} \leq \mathrm{M} \tag{33}$$

By substituting $\theta = \alpha\left(\lambda - \lambda X(z)\right)$ in the equations (29) , (30) and (31), we get

$$C(z,0) = \tilde{C}\left(\alpha\left(\lambda_0 - \lambda_0 X(z)\right)\right)\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{m,n}(0)\, z^n\,; \tag{34}$$

$$P_i(z,0) = \tilde{S}\left(\alpha\left(\lambda - \lambda X(z)\right)\right)\left[\sum_{m=a}^{b} P_{m,i}(0) + R_i(0) + \sum_{l=1}^{M} Q_{l,i}(0) + \sum_{m=0}^{a-1} T_m \lambda g_{i-m}\right]$$
$$\mathrm{a} \leq \mathrm{i} \leq \mathrm{b}-1 \tag{35}$$

$$P_b(z,0) = \frac{\tilde{S}\left(\alpha\left(\lambda - \lambda X(z)\right)\right) f(z)}{Z^b - \tilde{S}\left(\alpha\left(\lambda - \lambda X(z)\right)\right)} \tag{36}$$

where

$$\mathrm{f(z)}= \sum_{m=a}^{b-1} P_m(z,0) - \sum_{m=a}^{b}\sum_{j=0}^{b-1} P_{m,j}(0) z^j + \lambda\left(T(z)X(z) - \sum_{m=0}^{a-1} T_m z^m \sum_{\mathrm{j}=1}^{\mathrm{b-m-1}} \mathrm{g_j z^j}\right)$$
$$+\sum_{l=1}^{M}\left(\tilde{Q}_l(z,0) - \sum_{j=0}^{b-1} Q_{l,j}(0) z^j\right) \tag{37}$$

Substituting the expressions for $P_m(z,0)$, $\mathrm{a} \leq \mathrm{m} \leq \mathrm{b}-1$ from (35) and $\tilde{Q}_l(z,0), 1 \leq \mathrm{l} \leq \mathrm{M}$ from (32) and (36) in $f(z)$

$f(z)$

$$= \tilde{S}\left(\alpha\left(\lambda - \lambda X(z)\right)\right)\left\{\begin{array}{c} \sum_{n=a}^{b-1}\left[\left[\sum_{m=a}^{b} P_{m,n}(0) + R_i(0) + \sum_{l=1}^{M} Q_{l,n}(0) + \sum_{m=0}^{a-1} T_m \lambda g_{n-m}\right]\right] - \sum_{m=a}^{b}\sum_{j=0}^{b-1} P_{m,j}(0) z^j \\ \tilde{V}\left(\beta\left(\lambda_0 - \lambda_0 X(z)\right)\right)\sum_{n=0}^{a-1}\left[\sum_{m=a}^{b} P_{m,n}(0) + \sum_{j=1}^{M} Q_{j,n}(0)\right] z^n \\ -\sum_{j=1}^{M}\sum_{n=0}^{b-1} Q_{j,n}(0) z^n \\ +\tilde{C}\left(\alpha\left(\lambda_0 - \lambda_0 X(z)\right)\right)\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{m,n}(0)\, z^n \\ +\lambda\left(T(z)X(z) - \sum_{m=0}^{a-1}\left(T_m z^m \sum_{j=1}^{b-m-1} g_j z^j\right)\right) \end{array}\right\}$$

From the equation (27) & (32), we have

$$\tilde{Q}_1(z,\theta) = \frac{\left(\tilde{V}\left(\beta\left(\lambda_0 - \lambda_0 X(z)\right)\right) - \tilde{V}(\theta)\right) C(z,0)}{\left(\theta - \beta\left(\lambda_0 - \lambda_0 X(z)\right)\right)} \quad (38)$$

From the equation (28) & (33), we have

(39)

$$\tilde{Q}_j(z,\theta) = \frac{\left(\tilde{V}\left(\beta\left(\lambda_0 - \lambda_0 X(z)\right)\right) - \tilde{V}(\theta)\right)\sum_{n=0}^{b}\sum_{j=1}^{M} Q_{j-1,n}(0) z^n}{\left(\theta - \beta\left(\lambda_0 - \lambda_0 X(z)\right)\right)}, 2 \le j \le M \quad (40)$$

From the equation (29) & (34), we have

$$\tilde{C}(z,\theta) = \frac{\left(\tilde{C}\left(\alpha\left(\lambda_0 - \lambda_0 X(z)\right)\right) - \tilde{C}(\theta)\right)\sum_{n=0}^{a-1}\sum_{j=1}^{M} P_{m,n}(0) z^n}{\left(\theta - \alpha\left(\lambda_0 - \lambda_0 X(z)\right)\right)}$$

From the equation (30) & (35), we have

$$\tilde{P}_i(z,\theta) = \frac{\left(\tilde{S}\left(\alpha\left(\lambda - \lambda X(z)\right)\right) - \tilde{S}(\theta)\right)\left[\sum_{m=a}^{b} P_{m,i}(0) + \sum_{l=1}^{M} Q_{l,i}(0) + \sum_{m=0}^{a-1} T_m \lambda g_{i-m}\right]}{\left(\theta - \alpha\left(\lambda - \lambda X(z)\right)\right)}$$

$a \le i \le b-1$ (41)

From the equation (31) & (36), we have

$$\tilde{P}_b(z,\theta) = \frac{\left[\tilde{S}\left(\alpha\left(\lambda - \lambda X(z)\right)\right) - \tilde{S}(\theta)\right] f(z)}{\left(\theta - \alpha\left(\lambda - \lambda X(z)\right)\right)\left(z^b - \tilde{S}\left(\alpha\left(\lambda - \lambda X(z)\right)\right)\right)} \quad (42)$$

Using the Equations (38) (39), (40) ,(41) and (42) in the Equation (26), the probability generating function of the queue size P(z) at an arbitrary time epoch is obtained as

$$P(z) = \frac{\left(\tilde{S}\left(\alpha\left(\lambda - \lambda X(z)\right)\right) - 1\right)\left[\sum_{m=a}^{b} P_{m,i}(0) + \sum_{l=1}^{M} Q_{l,i}(0) + \sum_{m=0}^{a-1} T_m \lambda g_{i-m}\right]}{\left(-\alpha\left(\lambda - \lambda X(z)\right)\right)}$$

$$
\begin{aligned}
&+\frac{\left[\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-1\right] f(z)}{\left(-\alpha\left(\lambda-\lambda X(z)\right)\right)\left(Z^b-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)}\\
&+\frac{\left(\tilde{V}\left(\beta\left(\lambda_0-\lambda_0 X(z)\right)\right)-1\right) C(z,0)}{\left(-\beta\left(\lambda_0-\lambda_0 X(z)\right)\right)}\\
&+\frac{\left(\tilde{V}\left(\beta\left(\lambda_0-\lambda_0 X(z)\right)\right)-1\right)\sum_{j=1}^{M} Q_{j-1,n}(0) z^n}{\left(-\beta\left(\lambda_0-\lambda_0 X(z)\right)\right)}+T(z)\\
&+\frac{\left(\tilde{C}\left(\alpha\left(\lambda_0-\lambda_0 X(z)\right)\right)-1\right)\sum_{n=0}^{a-1}\sum_{m=a}^{b} P_{m,n}(0)\, z^n}{\left(-\alpha\left(\lambda_0-\lambda_0 X(z)\right)\right)}
\end{aligned}
\tag{43}
$$

Let

$p_i=\sum_{m=a}^{b} P_{m,i}(0)$, $r_i= R_i(0), q_i=\sum_{l=1}^{M} Q_{l,i}(0)$ and $c_i=p_i+q_i+r_i$ (44)

Simplifying eqution (43) by using (44), we have

$$
P(z)=\frac{\left\{\begin{array}{c}
\beta\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-1\right)\left(-\lambda_0+\lambda_0 X(z)\right)\sum_{i=a}^{b-1} c_i\left(z^b-z^i\right)\\
+\left(\tilde{V}\left(\beta\left(\lambda_0-\lambda_0 X(z)\right)\right)-1\right)\left[\beta\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-1\right)\tilde{C}\left(\beta\left(\lambda_0-\lambda_0 X(z)\right)-1\right)\left(-\lambda_0+\lambda_0 X(z)\right)+\alpha\left(Z^b-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)\left(-\lambda+\lambda X(z)\right)\right]\sum_{i=0}^{a-1} c_i z^i\\
-\left(\tilde{V}\left(\beta\left(\lambda_0-\lambda_0 X(z)\right)\right)-1\right)\left[\beta\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-1\right)\left(-\lambda_0+\lambda_0 X(z)\right)+\alpha\left(Z^b-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)\left(-\lambda+\lambda X(z)\right)\right]\sum_{i=0}^{a-1} p_i z^i\\
+\beta\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-1\right)\left(-\lambda_0+\lambda_0 X(z)\right)\sum_{i=a}^{b-1}\left(z^b-z^i\right)\sum_{m=0}^{a-1} T_m \lambda g_{i-m}+a_1\beta\lambda T(z)\left(-\lambda_0+\lambda_0 X(z)\right)\left(X(z)-1\right)\left(z^b-1\right)
\end{array}\right\}}{\alpha\beta\left(-\lambda+\lambda X(z)\right)\left(-\lambda_0+\lambda_0 X(z)\right)\left(Z^b-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)}
\tag{45}
$$

The probability generating function P(z) has to satisfy P(1) = 1.
In order to satisfy the condition, applying L'Hospital's rule and evaluating $\lim_{z\to\infty}$ P(z) and equating the expression to 1, $b-\lambda\alpha E(X)[E(S)]>0$ is obtained.

Define 'ρ' as $\frac{\lambda\alpha E(X)[E(S)]}{b}$. Thus ρ<1 is the condition to be satisfied for the existence of steady state for the model.

V. Particular case

(i). When the number of vacations become infinite , then M=∞ and the equation (45)reduces

$$
P(z)=\frac{\begin{array}{c}
\beta\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-1\right)\left(-\lambda_0+\lambda_0 X(z)\right)\sum_{i=a}^{b-1} c_i\left(z^b-z^i\right)\\
+\left(\tilde{V}\left(\beta\left(\lambda_0-\lambda_0 X(z)\right)\right)\right)\left[\beta\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)\tilde{C}\left(\beta\left(\lambda_0-\lambda_0 X(z)\right)-1\right)\left(-\lambda_0+\lambda_0 X(z)\right)+\alpha\left(Z^b-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)\left(-\lambda+\lambda X(z)\right)\right]\sum_{i=0}^{a-1} c_i z^i
\end{array}}{\alpha\beta\left(-\lambda+\lambda X(z)\right)\left(-\lambda_0+\lambda_0 X(z)\right)\left(Z^b-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)}
\tag{46}
$$

Equation(46) gives the PGF of queue length distribution of an M^x /G(a,b)/1 with state -dependent arrivals and multiple vacations.The result coincides with queue length distribution of Arumuganathan and Ramaswami(2005)

When the closedown time is one and the equation (46) reduces

$$
P(z)=\frac{\begin{array}{c}
\beta\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)-1\right)\left(-\lambda_0+\lambda_0 X(z)\right)\sum_{i=a}^{b-1} c_i\left(z^b-z^i\right)\\
+\left(\tilde{V}\left(\beta\left(\lambda_0-\lambda_0 X(z)\right)\right)\right)\left[\beta\left(\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)\left(-\lambda_0+\lambda_0 X(z)\right)+\alpha\left(Z^b-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)\left(-\lambda+\lambda X(z)\right)\right]\sum_{i=0}^{a-1} c_i z^i
\end{array}}{\alpha\beta\left(-\lambda+\lambda X(z)\right)\left(-\lambda_0+\lambda_0 X(z)\right)\left(Z^b-\tilde{S}\left(\alpha\left(\lambda-\lambda X(z)\right)\right)\right)}
\tag{47}
$$

Equation(46) gives the PGF of queue length distribution of an M^x /G(a,b)/1 with state -dependent arrivals and multiple vacations.The result coincides with queue length distribution of Arumuganathan and jayakumar.

VI. Conclusion

In this paper, a bulk arrival general bulk service queuing system with modified M-vacation policy, variant arrival rate under a restricted admissibility policy of arriving batches and close down time is considered. Probability generating function of queue size at an arbitrary time epoch and particular case are obtained.

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