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Signed Total Roman Dominating Functions of Rooted Product Graphs

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ABSTRACT: *In this paper, we study the signed roman dominating functions, signed total roman dominating functions of rooted product graph $G = P_n o C_m$, where P_n be a Path graph with n vertices and $C_m (m \geq 3)$ be a cycle with a sequence of n rooted graphs $C_{m1}, C_{m2}, \dots\dots, C_{mn}$. Also we check the minimality of the signed roman(signed total roman) dominating functions.*

KEYWORDS: *Rooted product graph, signed roman dominating functions, signed total roman dominating functions.*

subject classification code: 05C10

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I. INTRODUCTION

Let $f: V \to \{-1,1,2\}$ be a function, is said to be a signed roman dominating function (SRDF) of G, if $f(N[v]) = \sum_{u \in N[v]} f(u) \geq 1$, for each $v \in V$ and satisfying the condition that every vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$. It is minimal signed roman dominating function (MSRDF), if for all $g < f$, g is not a SDF. The weight of f is the sum of the function value of all vertices in G, i.e., $f(V(G)) = \sum_{u \in V(G)} f(u)$. The signed roman domination number of G, $\gamma_{sR}(G)$**,** is the minimum weight of a SRDF of G.

A function $f: V \to \{-1,1,2\}$ is called a signed total roman dominating function of G, if $f(N(v)) = \sum_{u \in N(v)} f(u) \geq 1$, for each $v \in V$ and satisfying the condition that every vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$. It is minimal signed total roman dominating function (MSTRDF), if for all $g < f$, g is not a STRDF. The weight of f is the sum of the function value of all vertices in G. The signed total roman domination number of G, $\gamma_{stR}(G)$**,** is the minimum weight of a STRDF of G.

In 1995 Dunbar, Hedetniemi, Henning and Slater [2] published the first paper entitled "Signed domination in graphs" and also referred in [3].

Volkmann [6,7] has studied about signed total roman domination in digraphs, signed total roman domination in graphs.

In 2014, Ahangar, Henning, Lowenstein, Zhao and Samodivkin [1] introduced the concept of signed roman domination in graphs.

A new product on two graphs G_1 and G_2 , called rooted product denoted by $G_1 \circ G_2$ and it was first introduced by Godsil and McKay [4] and also we refered in [5].

II. RESULTS ON SIGNED ROMAN DOMINATING FUNCTIONS

In this section we can derived some results on the signed roman dominating functions of $G = P_n o C_m$.

Theorem 2.1: If the function $f: V \to \{-1,1,2\}$ is defined by

$$f(v) = \begin{cases} 2, if\ v = u_{ij} \in C_m\ and\ j \equiv 2 (mod\ 3) in\ each\ copy\ of\ C_m, \\ -1, if\ v = u_{ij} \in C_m\ and\ j \equiv 1 (mod\ 3) in\ each\ copy\ of\ C_m, \\ \qquad +1, \quad otherwise. \end{cases}$$

Then f is a minimal signed roman dominating function of $G = P_n o C_m$ and signed roman domination number of G is $\gamma_{sR}(G) = \frac{2mn}{3}$, when m is divisible by 3 in G.

Proof: Consider the rooted product graph $G = P_n o C_m$.

Let f be a function defined in the hypothesis.

Here -1 is assigned to $\frac{m}{3}$ vertices in each copy of C_m in G, 2 is assigned to $\frac{m}{3}$ vertices in C_m, and $+1$ is assigned to all other vertices in G.

Case 1: Suppose $v \in P_n$.
(i) As $d(v) = 4$ in G. Thus $\sum_{u\in N[v]} f(u) = [2 + (-1)] + [1 + 1 + 1] = 4$.
(ii) As $d(v) = 3$ in G. Thus $\sum_{u\in N[v]} f(u) = [(-1) + 2] + [1 + 1] = 3$.
Case 2: Suppose $v \in C_m$ be such that $d(v) = 2$ in G & $f(v) = -1, +1$ or 2.
Thus $\sum_{u\in N[v]} f(u) = [(-1) + 1 + 2] = 2$.
In both cases, we get $\sum_{u\in N[v]} f(u) \geq 1, \forall v \in V$.
This implies that f is a signed roman dominating function (SRDF).

Now $$\sum_{u\in N[v]} f(u) = \left(\underbrace{1+1+---+1}_{n-times}\right) + \left(\underbrace{\frac{m}{3}(-1)}_{n-times}\right) + \left(\underbrace{\frac{m}{3}(+2)}_{n-times}\right) + \underbrace{\left((m-1)-\frac{2m}{3}\right)(+1)}_{n-times} = \frac{2mn}{3}.$$

By the definition of signed roman domination number, $\gamma_{sR}(G) \leq \frac{2mn}{3} \to (1)$
Now we claim that f is a minimal signed roman dominating function.
For this we define $g: V \to \{-1,1,2\}$ by

$$g(v) = \begin{cases} -1, \text{ if any one vertex } v_k \in P_n, \\ 2, \text{ if } v = u_{ij} \in C_m \text{ and } j \equiv 2 \pmod 3 \text{ in each copy of } C_m, \\ -1, \text{ if } v = u_{ij} \in C_m \text{ and } j \equiv 1 \pmod 3 \text{ in each copy of } C_m, \\ +1, \text{ otherwise.} \end{cases}$$

Since, at the vertex $v_k \in P_n$ the strict inequality holds, it follows that $g < f$. Here we discuss about the condition $v_k \in N[v]$ and $v_k \notin N[v]$ is discussed in the above cases.
Case 3: Suppose $v \in P_n$.
(i) As $d(v) = 4$ in G, then $\sum_{u\in N[v]} g(u) = [2 + (-1)] + [1 + (-1) + 1] = 2$.
(ii) As $d(v) = 3$ in G, then $\sum_{u\in N[v]} g(u) = [2 + (-1)] + [(-1) + 1] = 1$.
Case 4: Suppose $v \in C_m$ be such that $d(v) = 2$ in G and $g(v) = -1$ or 2.
If $g(v) = -1$ then $\sum_{u\in N[v]} g(u) = (-1) + [2 + (-1)] = 0$.
If $g(v) = 2$ then $\sum_{u\in N[v]} g(u) = 2 + [(-1) + (-1)] = 0$.
From the above cases, we get $\sum_{u\in N[v]} g(u) < 1, for\ some\ v \in V$.
This implies that g is not a SRDF.
Hence f is a minimal signed roman dominating function of G.
Therefore for any signed roman dominating function f, $\sum_{u\in N[v]} f(u) \geq \frac{2mn}{3}$.
Thus $\gamma_{sR}(G) \geq \frac{2mn}{3} \to (2)$
From the above two inequalities (1) & (2), we get $\gamma_{sR}(G) = \frac{2mn}{3}$.
For example, the functional values are given at each vertex of the graph $G = P_4 o C_9$.

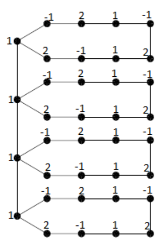

Figure: 1

Corollary 2.2 For any G, $\gamma_{\times 2}(G) = \gamma_R(G) = \gamma_{sR}(G)$ when m is divisible by 3 in G.
Proof: From reference[5] theorem 3.4.2, $\gamma_{\times 2}(G) = \frac{2mn}{3}$, theorem 4.3.1, $\gamma_R(G) = 2n\left(\frac{m}{3}\right)$ and
By the above theorem 2.1, $\gamma_{sR}(G) = \frac{2mn}{3}$. Clearly it follows that, $\gamma_{\times 2}(G) = \gamma_R(G) = \gamma_{sR}(G)$.
Corollary 2.3: For any G, $\gamma_{sR}(G) = 2\gamma_s(G)$ when m is divisible by 3 in G.
Proof: From reference[5] theorem 4.2.1, $\gamma_s(G) = \frac{mn}{3}$ and by the above theorem 2.1,
$\gamma_{sR}(G) = \frac{2mn}{3}$.
Clearly it follows that, $\gamma_{sR}(G) = 2\gamma_s(G)$.

Theorem 2.4: If the function $f: V \rightarrow \{-1,1,2\}$ is defined by

$$f(v) = \begin{cases} 2, if\ v = u_{ij} \in C_m\ and\ j \equiv 2(mod\ 3) in\ each\ copy\ of\ C_m, \\ -1, if\ v = u_{ij} \in C_m\ and\ j \equiv 1(mod\ 3) in\ each\ copy\ of\ C_m, \\ \quad +1, \quad otherwise. \end{cases}$$

Then f is a minimal signed roman dominating function of a graph G and signed roman domination number of G is $\gamma_{sR}(G) = n\left[m - \left\lfloor\frac{m}{3}\right\rfloor\right]$, for $m = 3\text{k} + 1$ in G.

Proof: Consider the graph $G = P_n o C_m$ with $|V|$ number of vertices and $|E|$ number of edges. Let f be a function defined in the hypothesis.

Here -1 is assigned to $\left\lfloor\frac{m}{3}\right\rfloor$ vertices in each copy of C_m in G, 2 is assigned to $\left\lfloor\frac{m}{3}\right\rfloor$ vertices in each copy of C_m, and +1 is assigned to all other vertices in G.

Case 1: Suppose $v \in P_n$.

(i) As $d(v) = 4$ in G, then $\sum_{u \in N[v]} f(u) = [1 + (-1)] + [1 + 1 + 1] = 3$.

(ii) As $d(v) = 3$ in G, then $\sum_{u \in N[v]} f(u) = [1 + (-1)] + [1 + 1] = 2$.

Case 2: Suppose $v \in C_m$ be such that $d(v) = 2$ in G then $f(v) = -1,2\ or + 1$.

If $f(v) = -1\ or\ 2$ then $\sum_{u \in N[v]} f(u) = [(-1) + 1 + 2] = 2$.

If $f(v) = +1$ then $\sum_{u \in N[v]} f(u) = [1 + 1 + 2] = 4$.

In the above cases f is a SRDF, because $\sum_{u \in N[v]} f(u) \geq v, \forall v \in V$.

This implies that f is a signed roman dominating function.

Now $$\sum_{u \in N[v]} f(u) = \left(\underbrace{1 + 1 + - - - + 1}_{n-times}\right) + \left(\underbrace{\left\lfloor\frac{m}{3}\right\rfloor(-1)}_{n-times}\right) + \left(\underbrace{\left\lfloor\frac{m}{3}\right\rfloor(+2)}_{n-times}\right) + \underbrace{\left((m-1) - 2\left\lfloor\frac{m}{3}\right\rfloor\right)(+1)}_{n-times} = n\left[m - \left\lfloor\frac{m}{3}\right\rfloor\right].$$

By the definition of signed roman domination number, $\gamma_{sR}(G) \leq n\left[m - \left\lfloor\frac{m}{3}\right\rfloor\right] \rightarrow (1)$

Now we check for minimality of f, define $g: V \rightarrow \{-1,1,2\}$ by

$$g(v) = \begin{cases} -1, \text{ if any vertex } v_k \in P_n, \\ 2, \text{ if } v = u_{ij} \in C_m \text{ and } j \equiv 2(\text{mod } 3) \text{ in each copy of } C_m, \\ -1, \text{ if } v = u_{ij} \in C_m \text{ and } j \equiv 1(\text{mod } 3) \text{ in each copy of } C_m, \\ +1, \text{ otherwise.} \end{cases}$$

Where $\text{i} = 1,2, - - -, \text{n}$.

Since, at the vertex $v_k \in P_n$ the strict inequality holds, it follows that $g < f$. Here we discuss about the condition $v_k \in N[v]$ and $v_k \notin N[v]$ is discussed in the above cases.

Case 3: Suppose $v \in P_n$.

(i) As $d(v) = 4$ in G, then $\sum_{u \in N[v]} g(u) = [1 + (-1)] + [1 + (-1) + 1] = 1$.

(ii) As $d(v) = 3$ in G, then $\sum_{u \in N[v]} g(u) = [1 + (-1)] + [(-1) + 1] = 0$.

Case 4: Suppose $v \in C_m$ be such that $d(v) = 2$ in G & $g(v) = -1, +1\ or\ 2$.

If $g(v) = -1$ then $\sum_{u \in N[v]} g(u) = (-1) + [2 + (-1)] = 0$.

If $\text{g}(v) = +1\ or\ 2$ then $\sum_{u \in N[v]} g(u) = (+1) + [2 + (-1)] = 2$.

This implies that g is not a SRDF, because $\sum_{u \in N[v]} g(u) < 1, for\ some\ v \in V$.

Hence f is a minimal signed roman dominating function on G.

Therefore for any signed roman dominating function f, $\sum_{u \in N[v]} f(u) \geq n\left[m - \left\lfloor\frac{m}{3}\right\rfloor\right]$.

Thus $\gamma_{sR}(G) \geq n\left[m - \left\lfloor\frac{m}{3}\right\rfloor\right] \rightarrow (2)$

From the above two inequalities (1) & (2), we get $\gamma_{sR}(G) = n\left[m - \left\lfloor\frac{m}{3}\right\rfloor\right]$.

Theorem 2.5: If the function $f: V \rightarrow \{-1,1,2\}$ is defined by

$$f(v) = \begin{cases} 2, if\ v = u_{ij} \in C_m\ and\ j \equiv 2(mod\ 3) in\ each\ copy\ of\ C_m, \\ -1, if\ v = u_{ij} \in C_m\ and\ j \equiv 1(mod\ 3) in\ each\ copy\ of\ C_m, \\ \quad +1, \quad otherwise. \end{cases}$$

The f is not a signed roman dominating function of $G = P_n o C_m$, for $\text{m} = 3\text{k} + 2$ in G.

Proof: Let f be a function defined in the hypothesis.

Here -1 is assigned to $\left\lceil\frac{m}{3}\right\rceil$ vertices in each copy of C_m in G, 2 is assigned to $\left\lceil\frac{m}{3}\right\rceil$ vertices in each copy of C_m, and $+1$ is assigned to other vertices in G.

Case 1: Suppose $v \in P_n$.
(i) As $d(v) = 4$ in G, then $\sum_{u \in N[v]} f(u) = [(-1)(2 - times)] + [1 + 1 + 1] = 1$.
(ii) As $d(v) = 3$ in G, then $\sum_{u \in N[v]} f(u) = [(-1)(2 - times)] + [1 + 1] = 0$.
Case 2: Suppose $v \in C_m$ be such that $d(v) = 2$ in G and $f(v) = -1, +1$ or 2.
Thus $\sum_{u \in N[v]} f(u) = [(-1) + 1 + 1] = 1$ or $\sum_{u \in N[v]} f(u) = [(-1) + 1 + 2] = 2$.
Since, $\sum_{u \in N[v]} f(u) < 1,$ $for\ some\ v \in V$. This implies that f is not a SRDF.

III. RESULTS ON SIGNED TOTAL ROMAN DOMINATING FUNCTIONS

In this section we can derived some results on the signed total roman dominating functions of $G = P_n o C_m$.
Theorem 3.1: If the function $f: V \to \{-1,1,2\}$ is defined by

$$f(v) = \begin{cases} 2, & \text{if } v = u_{ij} \in C_m \text{ and } j \equiv 1 \text{ or } 2 \pmod 3 \text{ in each copy of } C_m, \\ -1, & \text{if } v = u_{ij} \in C_m \text{ and } j \equiv 0 \pmod 3 \text{ in each copy of } C_m, \\ -1, & \text{if } v \in P_n. \end{cases}$$

Then f is a minimal signed total roman dominating function of $G = P_n o C_m$ and signed total roman domination number of G is $\gamma_{stR}(G) = mn$, when m is divisible by 3.
Proof: Suppose m is divisible by 3 and $m > 3$. Consider the graph $G = P_n o C_m$.
Let f be a function defined in the hypothesis.
In this graph, -1 is assigned to $\left(\frac{m}{3} - 1\right)$ vertices in each copy of C_m in G, 2 is assigned to $\frac{2m}{3}$ vertices of C_m, and -1 is assigned to all vertices of P_n.
Then by the definition of the function.
$f(u_{i1}) = 2, f(u_{i2}) = 2, f(u_{i3}) = -1,$
$f(u_{i4}) = 2, f(u_{i5}) = 2, f(u_{i6}) = -1,$
… … … … …
$f\left(u_{i(m-3)}\right) = -1, f\left(u_{i(m-2)}\right) = 2, f\left(u_{i(m-1)}\right) = 2.$
$And\ f(v_1) = f(v_2) = \cdots = f(v_n) = -1.$
Case 1: Suppose $v \in P_n$.
(i) As $d(v) = 4$ in G, then $N(v)$ contains two vertices of C_m and two vertices of P_n in G. Thus $\sum_{u \in N(v)} f(u) =$ *22−times+[(−1)(2−times)]=2.*
(ii) As $d(v) = 3$ in G, then $N(v)$ contains two vertices of C_m and one vertex of P_n in G. Thus $\sum_{u \in N(v)} f(u) =$ *22−times+(−1)=3.*
Case 2: Suppose $v \in C_m$ be such that $d(v) = 2$ in G then $f(v) = -1$ or 2.
If $f(v) = 2$ then $\sum_{u \in N(v)} f(u) = (-1) + (2) = 1$
And if $f(v) = -1$ then $\sum_{u \in N(v)} f(u) = (2) + (2) = 4$.
From the above cases, we get $\sum_{u \in N(v)} f(u) \geq 1, \forall v \in V$.
It follows that f is a STRDF.

$$\text{Now} \sum_{u \in N(v)} f(u) = \left(\underbrace{\left(\frac{m}{3}\right)(-1)}_{n-times} \right) + \left(\underbrace{\left(m - \frac{m}{3}\right)(+2)}_{n-times} \right) + \underbrace{(0)(+1)}_{n-times} = mn\ .$$

By the definition of signed total roman domination number, $\gamma_{stR}(G) \leq mn \to (1)$.
Now the minimality check for f, define $g: V \to \{-1,1,2\}$ by

$$g(v) = \begin{cases} 2, & \text{if } v = u_{ij} \in C_m \text{ and } j \equiv 1 \text{ or } 2 \pmod 3 \text{ in each copy of } C_m, \\ -1, & \text{if } v = u_{ij} \in C_m \text{ and } j \equiv 0 \pmod 3 \text{ in each copy of } C_m, \\ +1, & \text{if } v = u_{ik} \in C_m \text{ in } i^{th} \text{ copy of } C_m, \\ +1, & \text{otherwise.} \end{cases}$$

Since, at the vertex $u_{ik} \in C_m$ the strict inequality holds, it follows that $g < f$. Here we discuss about the condition $u_{ik} \in N(v)$ and $u_{ik} \notin N(v)$ is discussed in the above cases.
Case 3: Suppose $v \in P_n$.
(i) As $d(v) = 4$ in G then $\sum_{u \in N(v)} g(u) = [(2) + (1)] + [(-1) + (-1)] = 1$.
(ii) As $d(v) = 3$ in G then $\sum_{u \in N(v)} g(u) = [(2) + (1)] + (-1) = 2$.

Case 4: Suppose $v \in C_m$ be such that $d(v) = 2$ in G.
Thus $\sum_{u \in N(v)} g(u) = (-1) + (1) = 0$.
From the above cases, we get $\sum_{u \in N(v)} g(u) < 1, for\ some\ v \in V$.
This implies that g is not a STRDF. Hence f is a minimal STRDF on G.
Therefore for any STRDF f, $f(V) = \sum_{u \in N(v)} f(u) \geq mn$.
Thus $\gamma_{stR}(G) \geq mn \rightarrow (2)$.
From the above two inequalities (1) &(2), we get $\gamma_{stR}(G) = mn$.
For example, the functional values are given at each vertex of the graph $G = P_4 o C_9$.

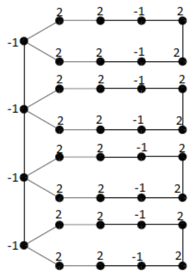

Figure:2

Corollary 3.2: For any G, $\gamma_{st}(G) = \gamma_{stR}(G)$ when m is divisible by 3 in G.
Proof: From reference[5] theorem 6.2.2, $\gamma_{st}(G) = mn$ and by theorem 3.1, $\gamma_{stR}(G) = mn$.
Clearly it follows that, $\gamma_{st}(G) = \gamma_{stR}(G)$.
Theorem 3.3: If the function $f: V \rightarrow \{-1,1,2\}$ is defined by

$$f(v) = \begin{cases} 2, & \text{if } v = u_{ij} \in \mathrm{C}_m \text{ and } j \equiv 1 \text{ or } 2 \pmod 3 \text{ in each copy of } \mathrm{C}_m, \\ -1, & \text{otherwise.} \end{cases}$$

Then f is not a signed total roman dominating function of $G = P_n o C_m$, when
$\mathrm{m} = 3\mathrm{k} + 1$ in G.
Proof: Suppose m is not divisible by 3. Consider the graph $G = P_n o C_m$ and f be a function defined in the hypothesis. Here -1 is assigned to $\left\lfloor \frac{m}{3} \right\rfloor$ vertices in each copy of C_m in G, 2 is assigned to $\left(m - \left\lfloor \frac{m}{3} \right\rfloor - 1\right)$ vertices of C_m, and -1 is assigned to all vertices of P_n.
Then by the definition of the function.
$f(u_{i1}) = 2, f(u_{i2}) = 2, f(u_{i3}) = -1,$
$f(u_{i4}) = 2, f(u_{i5}) = 2, f(u_{i6}) = -1,$
… … … … …
$f\left(u_{i(m-3)}\right) = 2, f\left(u_{i(m-2)}\right) = 2, f\left(u_{i(m-1)}\right) = -1.$
$And\ f(v_1) = f(v_2) = \cdots = f(v_n) = -1.$
Case 1: Suppose $v \in P_n$.
(i) As $d(v) = 4$ in G, then $N(v)$ contains two vertices of C_m and two vertices of P_n in G. Thus $\sum_{u \in N(v)} f(u) =$ $-12-times+-1+2=-1.$
(ii) As $d(v) = 3$ in G, then $N(v)$ contains two vertices of C_m and one vertex of P_n in G. Thus $\sum_{u \in N(v)} f(u) =$ $2+-1+-1=0.$
Case 2: Suppose $v \in C_m$ be such that $d(v) = 2$ in G then $f(v) = -1\ or\ 2$.
If $f(v) = 2$ then $\sum_{u \in N(v)} f(u) = (-1) + (2) = 1$.
If $f(v) = -1$ then $\sum_{u \in N(v)} f(u) = (2)(2 - times) = 4$.
Since, $\sum_{u \in N(v)} f(u) < 1, for\ some\ v \in V$. This implies that f is not a STRDF.
Theorem 3.4: If the function $f: V \rightarrow \{-1,1,2\}$ is defined by

$$f(v) = \begin{cases} 2, & \text{if } v = u_{ij} \in \mathrm{C}_m \text{ and } j \equiv 1 \text{ or } 2 \pmod 3 \text{ in each copy of } \mathrm{C}_m, \\ -1, & \text{otherwise.} \end{cases}$$

Then f is not a signed total roman dominating function of $G = P_n o C_m$, for
$\mathrm{m} = 3\mathrm{m} + 2$ in G.

Proof: Suppose m is not divisible by 3. Consider the graph $G = P_n o C_m$ and f be a function defined in the hypothesis. Here -1 is assigned to $\left\lfloor \frac{m}{3} \right\rfloor$ vertices in each copy of C_m in G, 2 is assigned to $\left(m - \left\lfloor \frac{m}{3} \right\rfloor - 1\right)$ vertices of C_m, and -1 is assigned to all vertices of P_n.
Then by the definition of the function.
$f(u_{i1}) = 2, f(u_{i2}) = 2, f(u_{i3}) = -1,$
$f(u_{i4}) = 2, f(u_{i5}) = 2, f(u_{i6}) = -1,$
… … … … …
$f(u_{i(m-3)}) = 2, f(u_{i(m-2)}) = 2, f(u_{i(m-1)}) = -1.$
$And\ f(v_1) = f(v_2) = \cdots = f(v_n) = -1.$
Case 1: Suppose $v \in P_n$.
(i) As $d(v) = 4$ in G, then $N(v)$ contains two vertices of C_m and two vertices of P_n in G. Thus $\sum_{u \in N(v)} f(u) =$ *22−times+−1(2−times)=2.*
(ii) As $d(v) = 3$ in G, then $N(v)$ contains two vertices of C_m and one vertex of P_n in G. Thus $\sum_{u \in N(v)} f(u) =$ *22−times+−1=3.*
Case 2: Suppose $v \in C_m$ be such that $d(v) = 2$ in G.
If $f(v) = f(u_{i1})$ then $\sum_{u \in N(v)} f(u) = (-1) + (2) = 1.$
If $f(v) = f(u_{i(m-1)})$ then $\sum_{u \in N(v)} f(u) = (-1) + (-1) = -2.$
And $f(v) = f(u_{ij})$ then $\sum_{u \in N(v)} f(u) = (-1) + (2) = 1$
$or\ \sum_{u \in N(v)} f(u) = (2)(2 - times) = 4$, here $j \neq 1\ or\ (m-1)$.
From the above cases, we get $\sum_{u \in N(v)} f(u) < 1, for\ some\ v \in V.$
This implies that f is not a signed total roman dominating function.
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