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Augmented Markov Chain Model for monitoring Tuberculosis Disease

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Abstract
The use of augmented markov chain model with control chart in health management processes is very scanty and few due to the belief of some researchers that control chart can only be applied to quality of production processes. Therefore, we aimed to apply control chart through augmented markov chain to health data on the application of DOTS to tuberculosis data. The augmented markov chain model used is $Pij = P\{Y_{n+1=j} | Y_n\}$, $n = 0,1,2..,i,j = 0,1$ *where,* P_{ij} *is the transition probability matrix,* Y_n *is the nth inspected patient and 0 and 1 are the success and failure state respectively. Based on the analysis conducted with data collected on 250 patients suffering from Tuberculosis. The initial frequencies of the two states of application are (Success = 45, Failure = 205). 25 transitions from success to success, 20 transitions from success to failure, 19 transitions from failure to success and 185 transitions from failure to failure. The probability of moving from success to success is 0.56, the probability of moving from success to failure is 0.44, and the probability of moving from failure to success is 0.09 while the probability of moving from failure to failure is 0.91. The probability of success is 0.18 while the probability of failure is 0.82. Therefore, the probability of success falls below the lower control limit (0.64). However, the system (application) is out of control. The application of DOTS in the treatment of tuberculosis patients has served as an efficient and effective tool for the classification of patients based on their observable states. Generally, the study shows that there is an increment in failure state of the application of DOTS on tuberculosis patients.*

Key words: *Markov chain, transition probability, initial state, DOTS, probability*

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I. INTRODUCTION

Nowadays some industries including health providers carry out measurement and evaluation so as to conform to specification. In many healthprocesses, inparticularinthe Chemicalindustry,it isnot possible to take more than one observationat each time ofsampling. In such cases a common practice is to consider a controlchart for individuals (X) so as to supervise processes. However,taking inaccountof performance ofthe monitoring schemeispositively correlatedwiththesamplesizetaken toinspection, the drawback ofhaving available onlyonenewobservationat each timeofsampling couldbecompensatedusingacontrolstatisticthataccumulatestheinformationof past data. Manypractitionersbegantousethe movingaverage (MA).General details about control charts may befound, forinstance, inRyan (2000) and in Montgomery (2005).Themostcommoncontrol schemesperformance measures arerelated withthe time ofthefirstpassage ofthecontrol statisticover a given threshold. These measures are usually determinedbyMonte Carlosimulations,because analyticalexpressionsareingeneraldifficult,orevenimpossible,tobederived. Forinstance, wenotethat consecutive valuesofCUSUM andE W M A control statisticsshareobservationsofdifferent samples,andthus, toevaluate theirperformance, wehavetoconsider the structureofthe data processtogetherwith the structure ofthe dependence betweenconsecutive values. This variable representsthelimits' controlofthechart; practically, weusually compute itsaverage runlength, theARL, andeventually,itsstandarddeviation. Tocompare differentcharts they must havethesamein-control ARL, whichis ingeneralalarge,pre-fixedvalue.The importanceofthe ARL as a performance measure cannot be over – emphasized.controlschemewithspecificproper- ties, leadustofindanalyticalexpressions fortheARL ofthe charts usually used inpractice. Somegenericexpressions allowusto obtain the ARL.Tomotivatetheuseofk-dependentmovingmaxima (MM^k)andmovingsum (MS^k)control charts,weadvance insection3withsomedistributionalpropertiesabout thesecontrol statistics;wealsopresent,fork≤3,explicit expressions fortheprobabilitiesthatareusedinthecomputationoftheARLoftheMM^k andof the MS^k charts, implementedunder the assumptionofindependentobservations fromnormal orexponentialprocesses.Letusgenerally denotethecontrolstatisticassociated toagivencontrolchart attime tbyW_t,t≥1.Wesaythatweareinapresenceofak-dependentstructure, withk≥1,ifforallt≥1wehave

$W_t$andW_{t+i}aredependentfori<k, $W_t$andW_{t+i}areindependentfori≥k.

Intheparticularcaseofk=1,thevariables $W_t$areindependentforeveryt.

Most parametric controlcharts can adapt toone or morecontrolparametersrelatively topre-fixedtargetsinboth directions (i.e.,are two–sidedcontrolcharts),and have the followingdecisionrule: ateachsampling pointtimet,thevaluesofthecontrolstatistic$W_t$should be evaluated with the limits.

Variation due to assignable factors can be identified and are responsible for important and large changes in the quality of the products. It is necessary to eliminate them as soon as they are discovered. Examples are bad handling of machine by operating mechanical fault in plants; excessive wear on the clotting tool etc. variation due to assignable factors can be controlled.

Now the question is how would an inspector of a company determine whether the change in quality is as a result of random causes or assignable causes? This is the more reason statistical quality controls established to answer such question especially in this time of great competition among manufacturing industries.

1.2 Statement of Problem

The control process regarding the quality of health management and service(s) has been compromised in many circles. Many health management outfits are still far behind in the use of the concept of the analyzing chart controls of their health systems. However, many researchers belief that, only quality of production process can be controlled/monitored by control chart. Meanwhile, in this research we apply/extend control chart using augmented markov chain model to health data and the process is classified as success (0) and failure (1).

1.3 Aim and Objectives

This study looks at statistical quality control in the analysis of univariate variable related to health process.

Other objectives are:

(1) Determine transitional state probabilities.

(2) Determine the distributional probability for use in determining the control limits.

(3) Improve existing statistical quality control in the health sector.

1.4 Significance of the study

- This study is of great importance and relevance to the health sector with respect to improved quality of their services.
- It will assist the health sector in conducting their continuous health investigations.
- It will undoubtedly health practitioners to know the abnormalities in products produced which will promote evolution of statistical quality control.
- Lastly, this research work, creates awareness for health care sector in applying statistical quality control techniques in their health care processes, showing them the benefits which they are going to derive from it and also to those who has been applying it, they should not relent in their efforts.

1.5 Scope of The Study

This project focuses on health sector concerned with the health management and services. This application of quality control is not for foods industries alone, but applicable to all other sectors such as health so as to maintain a measure of quality in the management of health processes.

1.6 Limitation of Study

There are some difficult (constrains) faced in carrying out this project work among which money spent, time consumption and unwillingness on the part of some health care service providers to assist in getting useful data.

1.7 Definition of Terms

These are the definition of terms that are likely to be used in this project work.

(1) ***Quality Control:*** This involves all activities geared towards maintaining quality.

(2) ***Quality:*** This is the conformance of a, product to the requirements with reference to what the consumers need.

(3) ***Customers:*** This is a person imparted by any process, which can either be internal or external

(4) ***Producer:*** This is a person that provides a process of goods and services which can be internal or external.

(5) ***Attribute Chart:*** A chart used in quality control to analyze a product and to determine if it is detective or acceptable.

(6) ***Process Chart:*** This is the method of maintaining the quality of a product coming out in a given process.

(7) ***Variation:*** this is the ability of finish goods to meet up normal specification or laid down standards of change in finish goods.
(8) ***Upper Control Limit (U.C.L):*** this is the maximum acceptable value of a mean, range etc. of a sample that is tested in a quality control study.
(9) ***Lower Control Limit (L.C.L):*** this is the minimum acceptable value if a mean, range etc. of a sample that is used in a quality control study.
(10) ***Change Variation:*** this is a random variation in manufactured item that cannot be eliminated entirely

II. LITERATURE REVIEW

Introduction

The concept of the literature review is that which gives an insight about the view of scholars, founder of statistical quality control and contribution of various authors. Statistical quality control is an important tool that has been existing since the early 1900's most especially during the third world war when the quality of bullets being used are to be tested, then the certain samples of the bullet were chosen and being shot to test the penetration power and the distance at which the bullet can move.

2.1 Markov Chain Model

Consideringrandomprocesses$\{X^{(n)}, n=0,1,2,...\}$which can be in a countable form of $p$if$j=i$where I takes values from 0 to N. This is the absorbing state.

2.2 An Augmented Markov Chain

Fu, Spiring and Xie (2001) applied different types of Markov chain on continuous system of variables. They were able to differentiate natural discretises and artificial discretises.Charts can be used to monitor different systems in counts or in continuous forms.They discovered that artificialsystems can be in form of EWMA charts. They ran a lengthy run bound to a countable Markov chain. Bai and Lee (2002) applied Markov chain in deriving formulae for evaluating signals.

Stounbos and Reynolds (2005) used probability approach in a false state of ARL to determine and control the applicability of system charts. It is required as a way of specifying certain calculable probabilities in distributive time shift.Yang (2005) posited that this models can be applied in economic systems to evaluate and projecteconomic values.De Magalhães, Costa and Moura (2006) measured some control charts of adaptive processes. Faraz and Saniga (2011) were able to make use of charts parachuted with double rules to exemplify the properties of T2 charts with Markov chains.

Yang and Rahim (2000) used Markov chain to extend economic evaluation using parameters of X and S to bring down high cost of processes. Yang and Chen (2011) applied the Markov chain in the derivation of average signal time in measuring how VSI works. Chang and Wu (2009) ran several length parameters using different control charts and discovered the auto – correlationality existing among them. Niaki, et al.(2010) examined ARL using investigative charts. Luma and Aparisi (2011) examined multiphaseMarkov chain approach in computing ARL and EWMA control algorithms.

III. METHODOLOGY

3.1 Data and Instrument

The data was extracted from a survey on"Appraisal of Treatment strategy of Tuberculosis Eradication of Secondary Healthcare Facility, Southwest, Nigeria" at West Africa Post Graduate College of Pharmacists, Yaba, Lagos.The sample size used for study was 250 TB patients.
The questionnaire contain 42 item which was divided into 4 subsections (A – D).

3.2 Method of Data Collection and Analysis

The data used for the project was collected from the individual attending the out – patients department of the hospital. Convenient sampling technique was used. The data was collected three times a week for the period one and a halved month. Excel and SPSS were used to analyse the data using Augmented Markov process and control chart

3.3 The Model:An Augmented Markov Chain

This is defined as a double state scenario of 0 and 1. Then

$$i, j = 0,1$$

$(p – k/\mathrm{Var}(X_n)/n, \quad p + k/\mathrm{Var}(X_n)/n)$
100(l-a)% Limits
(IL/n, IU/n)

$$\sum_{h=0}^{h=IL} P(X_n = h) \geq a/2$$

where

and

$$\sum_{h=0}^{h=IU} P(X_n = h \geq 1 = \alpha/2$$

Data Analysis

Results are presented using tabular presentations. Data were collected on 250 patients suffering from Tuberculosis. There are two possible states in the Markov chain, which are 0 and 1. States 0 and 1 represent success and failure respectively.

Table 1: Sequence of observations

Data

0,1,1,1,1,1,1,1,0,1,0,0,0,0,0,0,0,0,1,1,1,1,1,1,1,0,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1, 1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1, 1,1,1,1,1,1,1,1,1,1,1,1,1,0,1,0,0,0,0,0,0,0,0,1,1,1,1,1,1,0,0,0,0,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,1,1,1,1,1,1,1,1,0,1,1,1, 1,1,1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,1,1,1,0,1,1,1,1,1,1,0,0,0,0,1,1,1,1,1,1,1, 1,1,1,1,1,1,1,1,1,1

Sample size (n) = 250

Total number of failure = 205

Total number of success = 45

failure (P) = 0.82

success (1-P) =0.18

The number of Y_{00} = 25

The number of Y_{01} = 20

The number of Y_{10} = 19

The number of Y_{11} = 185

These are the initial frequencies of the two states of application (Success = 45, Failure = 205). There were 25 transitions from success to success, 20 transitions from success to failure, 19 transitions from failure to success and 185 transitions from failure to failure.

Table 2: The probability transition matrix

The transition probability matrix P_{ij} is derived as

$$P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix} = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} = \begin{bmatrix} 0.5556 & 0.4444 \\ 0.0931 & 0.9069 \end{bmatrix}$$

The probability of moving from success to success is 0.56, the probability of moving from success to failure is 0.44. The probability of moving from failure to success is 0.09 while the probability of moving from failure to failure is 0.91.

ρ= 1 – (a + b) = 1 – (0.4444 + 0.0931) = 0.4625. The value of the serial correlation between the two states is 0.46. This implies that there is a low positive correlation between the two states.

$$(\pi_0, \pi_1) = \left(\frac{b}{a+b}, \frac{a}{a+b}\right) = \left(\frac{0.0931}{0.4444 + 0.0931.}, \frac{0.4444}{0.4444 + 0.0931}\right) = (0.1732{,}0.8268)$$

The table 3 below shows the state (failure = 1 or success = 0), number of failures in n inspected persons, and the probability of failures in n inspected persons

Table 3:

Patients	Y_n	X_n	$P(X_n)$
1	0	0	0
2	1	1	0.5
3	1	2	0.666667
4	1	3	0.75
5	1	4	0.8
6	1	5	0.83333
7	1	6	0.85714
8	1	7	0.875
9	0	7	0.77778
-	-	-	-
-	-	-	-
249	1	204	0.819277

250	1	205	0.82

The cumulative probability of failure is 0.82 for all the patients.

$$Var(X_n) = 99.85312$$

Also, the standard deviation is computed as

$Sd(X_n) = \sqrt{Var(X_n)} = 9.992653$

$$UCL = p + k\sqrt{\frac{Var(X_n)}{n}} = 1.00$$

$$LCL = p - k\sqrt{\frac{Var(X_n)}{n}} = 0.64$$

Where k is the 95% confidence level

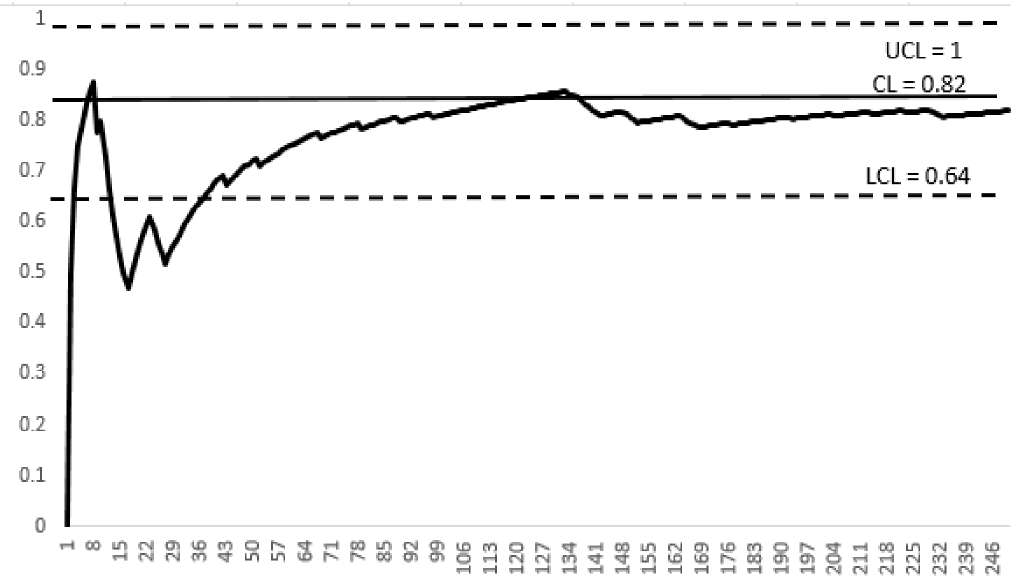

Figure 1: *Control Chart for mean Fraction of Failure*

This chart indicates that patients 15 to 36 are below lower control limit (0.64). There was an increase between 99 to 127 which was slightly above the central limit (0.82) i.e number of failure increased while, there was a decrease from 137 which was below the control limit (0.82).

IV. DISCUSSION OF FINDINGS

Based on the analysis conducted on the data collected from 250 patients suffering from Tuberculosis, the initial frequencies of the two states of application were (Success = 45, Failure = 205). There were 25 transitions from success to success, 20 transitions from success to failure, 19 transitions from failure to success and 185 transitions from failure to failure. The probability of moving from success to success was 0.56, the probability of moving from success to failure was 0.44, and the probability of moving from failure to success was 0.09 while the probability of moving from failure to failure was 0.91. The chart above indicates that patients 15 to 36 are below lower control limit (0.64). There was an increase between 99 to 127 which was slightly above the central limit (0.82) i.e. number of failure increased while, there was a decrease from 137 which was below the control limit (0.82).

4.2 Contribution to Knowledge

In the past, application of Augmented Markov Dependent control Chart is common to production processes (defective and non-defective products) in literature: and scanty to other areas.

However in this project, we have been able to use Augmented Markov Dependent Control Chart to:

1. Determine the transition probability of the application of DOTS on tuberculosis patients and we realized that the probability of moving from success to success was 0.56, the probability of moving from success to failure

was 0.44, and the probability of moving from failure to success was 0.09 while the probability of moving from failure to failure was 0.91.
2. We found limits of control with 1as higher limit, 0.82 at centre0.64 as the lower control limit.
3. We discovered that the probability of success (0.18) was below the lower control limit of 0.64 which makes the application of DOTS in the treatment of tuberculosis patients out of control. This model: "The Augmented Markov Chain" can equally be applied to various sectors apart from health.

V. FINDINGS, CONCLUSION AND RECOMMENDATION

5.0 Preamble

The importance of "Appraisal strategies and Tuberculosis Eradication cannot be overemphasized. Every health organization requires patients to get healed after medication, it is not enough to just have patients, health organizations must be able to manage their patients and solve their immediate health challenges. Markov Chain Control Chart is employ to show the true appraisal state of a health facility in Lagos State with respect to two appraisal conditions success (0) or failure (1) and then obtained the control limits of appraisal states of patients through the observed sequence of appraisals made over a period of time.

5.1 Summary of Findings

Data were collected on 250 patients suffering from Tuberculosis. The probability of moving from success to success was 0.56, the probability of moving from success to failure was 0.44, and the probability of moving from failure to success was 0.09 while the probability of moving from failure to failure was 0.91. The success rate was 0.18 while failure was 0.82. However, the system (application) is out of control. Figure 1 indicates that the chart that patients 15 to 36 were below lower control limit (0.64). There was an increase between 99 to 127 which was slightly above the central limit (0.82), i.e. number of failure increased while, there was a decrease from 137 which was below the control limit (0.82).

5.2 Conclusion

The Markov Model Control Chart has helped in obtaining vital information on the application of DOTS in the treatment of tuberculosis patients and has served as an efficient and effective tool for the classification of patients based on their observable states. Generally, the study shows that there is an increment in failure state of the application of DOTS on tuberculosis patients. The information gotten from this model can be used by the health organization to enable them operate maximally in combating the menace and deadly effect of tuberculosis. DOTS can be applied in the treatment of patients suffering from tuberculosis selected for the study failed as can be seen in the finding.

5.3 Recommendation

The empirical application in this study involved two observable states. A worthwhile application would be to use the approach and estimation procedures with more than two observable states. The data used in this study was from a Secondary Healthcare Facility in Southwest, Nigeria" for 6 months. A study can be done with a richer data containing more patients and the time period extended to a year or more.

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