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Laboratorytestingresultstoproblemsofincompletelyobserved modelingdynamicsforinitial-boundarydistributedspatial-temporalprocessesperturbations

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ABSTRACT– *The differential model of system`s solution is constructed, which according to the root-meansquare criterion is agreed with present initial-boundary observations for its condition independently upon quantity and quality of the latter. The cases of discretely and infinitely defined observations are considered. The accuracy and uniqueness of the problem`s solution is evaluated. Suggested methodologies' practical implementation is conducted, by which is proved its simplicity, reliability and effectiveness.*

KEYWORDS–*systems with distributed parameters, initial-boundary problems, systems with uncertainness, incorrect problems, programmed-modeling complexes*

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I. INTRODUCTION

The questions of the study of the dynamics of spatial-temporal systems are complex both in theoretical study and in practical implementations. Problems arise both at the stage of mathematical mode process and at the stages of its mathematical study. These problems are compounded by the solution of specific applied problems for systems, parameters of which are not available for observation in the scale required for the authentic and reliable operation of the corresponding mathematical algorithms. An example of such systems is the distributed spatial-temporal processes, the dynamics of which is described by a differential model with incomplete quantity and quality, defined by the initial-boundary condition. Initiated in [1] and developed in [2, 3] the approach to modeling the dynamics of these systems allows to construct mathematical solution of differential equation model, which according to the root-mean square criterion satisfies thenatural quantity of initial-boundary conditions. The results of the laboratory testing outlined in [2, 3] of the researched methods of the considered processes are discussed below. The problems of constructing the condition of spatial-temporal systems defined in limited partly and unlimited spatial-temporal domains, subdivisions of which allow continuous or discrete observations on initial-boundary condition. In this case the issues of software implementation of algorithms for solving applied problems draw particular attention.

Research problems of incompletely observed spatially distributed dynamic systems

Let'sconsiderspatial-temporalprocess,function$y(s)$ofcondition of which inspatial-temporaldomain $S_0^T=\{s=(x,t):\ x\in S_0\subset R^n,\ 0\le t\le T\}$ isdeterminedbytheequation

$$L(\partial_s)y(s)=u(s), \tag{1}$$

where$L(\partial_s)$isalineardifferentialoperator,inwhich$\partial_s=(\partial_x,\partial_t)=(\partial_{x1},\dots,\partial_{x_n},\partial_t)andu(s)$isthefunctionofthedistributedexternal-dynamicperturbationswhichsupplythisprocess.

Wewillproceedfromthefact,thatthestudiedprocessallowsobservationforthefunction$y(s)$ofthesystemcond itionin:
discreteform

$$L_r^0(\partial_t)y(s) = Y_{rl}^0\Big|_{t=0,\,x=x_l^0} \qquad (r=\overline{1,R_0};\, l=\overline{1,L_0}), \tag{2}$$

$$L_\rho^\Gamma(\partial_x)y(s)\Big|_{x=x_l^\Gamma} = Y_{\rho l}^\Gamma \qquad (\rho=\overline{1,R_\Gamma};\, l=\overline{1,L_\Gamma}), \tag{3}$$

orcontinuousform

$$L_r^0(\partial_t)y(s)\big|_{t=0} = Y_r^0(x) \qquad (r=\overline{1,R_0};\, x\in S), \tag{4}$$

$$L_\rho^\Gamma(\partial_x)y(s)\Big|_{x=x^\Gamma} = Y_\rho^\Gamma(x^\Gamma,t) \qquad (\rho=\overline{1,R_\Gamma};\, t\in[0,T]). \tag{5}$$

Here$S\subset S_0$,$\Gamma\subset\Gamma_0$(Γ_0–outlineofthespatialdomainS_0), $x^\Gamma\in\Gamma$,$x_l^0\epsilon\ S$, $S_l^\Gamma\epsilon\Gamma\times[0,T]andL_r^0(\partial_t)$also$L_p^\Gamma(\partial_x)$–lineardifferentialoperators,number$R_0$and$R_\Gamma$ofwhich,asthenumberofinitial-boundaryconditions(2),(3)and(4),(5)inthegeneralcaseisn'tagreedwiththeorderofthedifferentialoperator$L(\partial_s)$onthe variables$x$and$t$.

Remainingwithinthemethodologicalframework,consideredin[2,3]thesolutionoftheinitial-boundaryproblems(1)–(3)and(1),(4),(5)issuchthat

a) $$\Phi_1=\sum_{r=1}^{R_0}\sum_{l=1}^{L_0}(L_r^0(\partial_t)y(s))\Big|_{t=0,\,x=x_l^0}-Y_{rl}^0)^2+\sum_{\rho=1}^{R_\Gamma}\sum_{l=1}^{L_\Gamma}(L_\rho^\Gamma(\partial_x)y(s))\Big|_{s=s_l^\Gamma}-Y_{\rho l}^\Gamma)^2\to\min_{y(s)}, \tag{6}$$

b) $$\Phi_2=\sum_{r=1}^{R_0}\int_S(L_r^0(\partial_t)y(s)\big|_{t=0}-Y_r^0(x))^2ds+\sum_{\rho=1}^{R_\Gamma}\int_{\Gamma\times[0,T]}(L_\rho^\Gamma(\partial_x)y(s)-Y_\rho^\Gamma(s))^2ds\to\min_{y(s)}, \tag{7}$$

wegiveintheform:

a) $$y(s)=y_\infty(s)+\int_{S^0}G(s-s')u_0(s')ds'+\int_{S^\Gamma}G(s-s')u_\Gamma(s')ds', \tag{8}$$

b) $$y(s)=y_\infty(s)+\sum_{m=1}^{M_0}G(s-s_m^0)u_m^0+\sum_{m=1}^{M_\Gamma}G(s-s_m^\Gamma)u_m^\Gamma, \tag{9}$$

where

$$y_\infty(s)=\int_{-\infty}^{+\infty}G(s-s')u(s')ds',$$

$$G(s-s')=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{1}{L(p)}e^{p(s-s')}dp. \tag{10}$$

Green'sfunctionof equation(1),inwhich $p=(\lambda,\mu)=(\lambda_1,...,\lambda_n,\mu),\ d\lambda=d\lambda_1,...,d\lambda_n d\mu,$ $p(s-s')=\sum_{i=1}^{n}\lambda_i(x_i-x_i')+\mu(t-t')$ .Representedbytheratio(8),(9)thefunctionofthecondition $y(s)$ equation(1)satisfiesaccurately[2]forand$u_0(s)$ $(s\in S^0=S_0\times(-\infty,0])$,$u_\Gamma(s)$ $(s\in S^\Gamma=(R^n\setminus S_0)\times[0,T])$ andthus $u_m^0=u_0(s_m^0)$, $u_m^\Gamma=u_\Gamma(s_m^\Gamma)$ .That'swhytheunknownfunctions$u_0(s)$,$u_\Gamma(s)$andtheirvalues $u_m^0\ \ (m=\overline{1,M_0}),\ u_m^\Gamma\ \ (m=\overline{1,M_\Gamma}),$ wedefinefromtheconditions(6)and(7)respectively.

Summary of the research problem's solutionof incompletely observed spatially distributed dynamic systems

Let'sconsidertheproblemsofsolutions' implementation(8),(9)ofproblems(1)–(3)and(1),(4),(5).Takingintoconsiderationthefact,thatthemethodologyofconstructingthefunction $G(s-s'),$ definedbytheratio(11),describedbyusin[4],werecordanalyticaldependenciesoffunctions$u_0(s)$, $u_\Gamma(s)$andvectors

$$u^0=col(u_m^0, m=\overline{1,M_0}),$$

$$u^\Gamma=col(u_m^\Gamma, m=\overline{1,M_\Gamma}),$$

whichareobtained[2, 3]inthesolutionofproblems(6)and(7)withtakingintoaccount(8).

Forproblems(6),(2),(3),(8)

$$u_0(s) = (A_{11}^T(s), A_{21}^T(s))P_1^+(Y + A_\upsilon) + \upsilon_0(s), \tag{11}$$

$$u_\Gamma(s) = (A_{12}^T(s), A_{22}^T(s))P_1^+(Y + A_\upsilon) + \upsilon_\Gamma(s), \tag{12}$$

where$\upsilon_0(s)$($s \in S^0$),$\upsilon_\Gamma(s)$($s \in S^\Gamma$)are arbitraryintegralschangesinthedomainofitsargumentfunction,

$$P_1 = \int_{(\bullet)} A(s)A^T(s)ds, \quad A_\upsilon = \int_{(\bullet)} A(s)\upsilon(s)ds,$$

$$A(s) = \begin{pmatrix} \left(A_{11}(s) \quad (s \in S^0)\right), & \left(A_{12}(s) \quad (s \in S^\Gamma)\right) \\ \left(A_{21}(s) \quad (s \in S^0)\right), & \left(A_{22}(s) \quad (s \in S^\Gamma)\right) \end{pmatrix}, \quad \upsilon(s) = \begin{pmatrix} \upsilon_0(s) \\ \upsilon_\Gamma(s) \end{pmatrix}, \quad Y = \begin{pmatrix} Y_0 \\ Y_\Gamma \end{pmatrix},$$

$$A_{1i}(s') = col((L_r^0(\partial_t)G(s - s')\Big|_{t=0,\, x=x_l^0},\, l = \overline{1, L_0}),\, r = \overline{1, R_0}),,$$

$$A_{2i}(s') = col((L_\rho^\Gamma(\partial_x)G(s - s')\Big|_{s=s_l^\Gamma},\, l = \overline{1, L_\Gamma}),\, \rho = \overline{1, R_\Gamma}) \quad (i = \overline{1, 2}),$$

$$Y_0 = col((Y_{rl}^0, l = \overline{1, L_0}), r = \overline{1, R_0}),$$

$$Y_\Gamma = col((Y_{\rho l}^\Gamma, l = \overline{1, L_\Gamma}),\, \rho = \overline{1, R_\Gamma}).$$

Forproblem(7),(4),(5),(9):

$$u^0 = (Q_{11}, Q_{12})(B_Y + \upsilon) + \upsilon_0, \tag{13}$$

$$u^\Gamma = (Q_{21}, Q_{22})(B_Y + \upsilon) + \upsilon_\Gamma, \tag{14}$$

whereυ_0,$\upsilon_\Gamma$arearbitraryintegralsinthedomainoftheargumentfunction,$M_0$and$M_\Gamma$aredimensionalvectors,

$$\begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} = P_2^+,\ \upsilon = \begin{pmatrix} \upsilon_0 \\ \upsilon_\Gamma \end{pmatrix},$$

$$P_2 = \int_{(\bullet)} B^T(s)B(s)ds, \quad B_Y = \int_{(\bullet)} B^T(s)Y(s)ds,$$

$$B(s) = \begin{pmatrix} B_{11}(x) \quad (x \in S), & B_{12}(x) \quad (x \in S) \\ B_{21}(s) \quad (s \in \Gamma \times [0, T]), & B_{22}(s) \quad (s \in \Gamma \times [0, T]) \end{pmatrix}, \quad Y(s) = \begin{pmatrix} Y^0(x) & x \in S \\ Y^\Gamma(s) & s \in \Gamma \times [0, T] \end{pmatrix},$$

$$B_{11}(x) = col(str(L_r^0(\partial_t)G(s - s_m^0)\Big|_{t=0}, m = \overline{1, M_0}), r = \overline{1, R_0}),$$

$$B_{12}(x) = col(str(L_r^0(\partial_t)G(s - s_m^\Gamma)\Big|_{t=0}, m = \overline{1, M_\Gamma}), r = \overline{1, R_0}),$$

$$B_{21}(s) = col(str(L_\rho^\Gamma(\partial_x)G(s - s_m^0), m = \overline{1, M_0}),\, \rho = \overline{1, R_\Gamma}),$$

$$B_{22}(s) = col(str(L_\rho^\Gamma(\partial_x)G(s - s_m^\Gamma), m = \overline{1, M_\Gamma}),\, \rho = \overline{1, R_\Gamma}),$$

$$Y^0(x) = col(Y_r^0(x), r = \overline{1, R_0}),$$

$$Y^\Gamma(s) = col(Y_\rho^\Gamma(s),\, \rho = \overline{1, R_\Gamma}).$$

Let'sconsiderthattheproblemsolution(6)issimplified,ifthefunction$y(s)$isgivenbytheratio(9).Inthiscase

$$u_0 = (A_{11}^T,\ A_{21}^T)P_1^+(\bar{Y} - Av) + v_0,\ \forall v_0 \in R^{M_0}, \tag{15}$$

$$u_\Gamma = (A_{12}^T,\ A_{22}^T)P_1^+(\bar{Y} - Av) + v_\Gamma,\ \forall v_\Gamma \in R^{M_\Gamma} \tag{16}$$

under

$$\bar{Y} = \begin{pmatrix} Y_0 \\ Y_\Gamma \end{pmatrix},\ A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, P_1 = AA^T,$$

$$Y_0 = \mathrm{col}((Y_{rl}^0 - L_r^0(\partial_t)y_\infty(s)\Big|_{\substack{t=0\\ x=x_l^0}},\ l=\overline{1,L_0}),\ r=\overline{1,R_0}),$$

$$Y_\Gamma = \mathrm{col}((Y_{\rho l}^\Gamma - L_\rho^\Gamma(\partial_x)y_\infty(s)\Big|_{x=x_l^\Gamma},\ l=\overline{1,L_\Gamma}),\ \rho=\overline{1,R_\Gamma}),$$

$$A_{11} = \mathrm{col}((\mathrm{str}(L_r^0(\partial_t)G(s-s_m^0)\Big|_{\substack{t=0\\ x=x_l^0}},\ m=\overline{1,M_0}),\ l=\overline{1,L_0}),\ r=\overline{1,R_0}),$$

$$A_{12} = \mathrm{col}((\mathrm{str}(L_r^0(\partial_t)G(s-s_m^\Gamma)\Big|_{\substack{t=0\\ x=x_l^0}},\ m=\overline{1,M_\Gamma}),\ l=\overline{1,L_0}),\ r=\overline{1,R_0}),$$

$$A_{21} = \mathrm{col}((\mathrm{str}(L_\rho^\Gamma(\partial_x)G(s-s_m^0)\Big|_{s=s_l^\Gamma},\ m=\overline{1,M_0}),\ l=\overline{1,L_\Gamma}),\ \rho=\overline{1,R_\Gamma}),$$

$$A_{22} = \mathrm{col}((\mathrm{str}(L_\rho^\Gamma(\partial_x)G(s-s_m^\Gamma)\Big|_{s=s_l^\Gamma},\ m=\overline{1,M_\Gamma}),\ l=\overline{1,L_\Gamma}),\ \rho=\overline{1,R_\Gamma})$$

and defined above Y vector.

Hereandthenbysigns"+"and $(\bullet)$ aredeterminedtheoperationsofpseudo-inversionmatrixandintegrationacrossthedomainofargumentschangeofmatrices$A(s)$,$B(s)$andvector$Y(s)$,$v(s)$functions.

Foundaccordingto(8),(9)withconsiderationof(11)–(16)function $y(s)$ is[2,3]thesolutionofequation(1).Theaccuracy,with whichthisfunctionsatisfiesconditions(2),(3)and(4),(5)isdeterminedbyvalues

$$\varepsilon_1^2 = \min_{y(s)} \Phi_1 = Y^T Y - Y^T P_1 P_1^+ Y,$$

$$\varepsilon_2^2 = \min_{y(s)} \Phi_1 = Y^T Y - Y^T P P^+ Y$$

forproblems((1)–(3),(8))and((1)–(3),(9))and

$$\varepsilon_3^2 = \min_{y(s)} \Phi_2 = \int Y^T(s)Y(s)ds - B_Y^T P_2^+ B_Y$$

forproblems((1),(4),(5),(9)),whereasmentionedabove,integrationintoisinthedomainofchangingtheargumentsofavectorfunction$Y(s)$.Theobtainedsolutionsareuniqueprovidingtheirsolutions $\lim_{N\to\infty}\det\left[A(s_i)A^T(s_j)\right]_{i,j=0}^{i,j=N} > 0,$

$\det P_1 > 0,$ $\det P_1 > 0$accordingly.

Theaboveanalyzedsolutionsoftheconsideredproblemshavealsothepossibilitywhenthespatialortemporaldomainsareunlimited(therearenoboundaries(3),(5)orinitial(2),(4)conditions)andtheboundaryorinitialconditionoftheprocessisnotavailableforobservations($R_0 = 0$або $R_\Gamma = 0$).The infiniteness of spatial-temporaldomainleadstothefact,thatin thedefined matrices functions$A(s)$,$B(s)$,vector$Y$andvector-functions$Y(s)$won't be available functions, blocks,vectorsandvector-functions $u_0(s)$, u^0, A_{11} $A_{11}(s)$, A_{21}, $A_{21}(s)$, $B_{11}(s)$, $B_{21}(s)$, Y_0, $Y^0(x)$ (withinfinitetemporalinterval),and $u_\Gamma(s)$, u^Γ, A_{12}, $A_{12}(s)$, A_{22}, $A_{22}(s)$, $B_{12}(s)$, $B_{22}(s)$, Y_Γ, $Y^\Gamma(s)$ (withinfinitespatial-domain).

Undertherestrictionofthespatial-temporaldomainandthelackofobservations(2),(4)and(3),(5)ontheinitialconditionandoutlineperturbationofthefunctionprocess $u_0(s), u_\Gamma(s)$ andvectors u^0, u^Γ inthedefinitions(8),(9)thefunctionsofcondition arestored.Inthementionedaboveratiosthefollowingcomponents A_{11}, $A_{11}(x)$, A_{12}, $A_{12}(x)$, Y_0, $B_{11}(x)$, $B_{12}(x)$, $Y_0(x)$ will be available (noobservationsoftheinitialconditionoftheprocess)and A_{21}, $A_{21}(s)$, A_{22}, $A_{22}(s)$, Y_Γ, $B_{21}(s)$, $B_{22}(s)$, $Y_\Gamma(s)$ (noobservationsoftheboundaryconditionoftheprocess).

Program testing of research problem's solutionof incompletely observed spatially distributed dynamic systems

The above mentioned solutions of the considered problems on constructing function $y(s)$ according to (8), (9) were carefully tested by students of the Faculty of Computer Science and Cybernetics of Kyiv National Taras Shevchenko University, National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute" and Vasil Stefanyk Precarpathian National University during the last 7–10 years.

Testing proved to be quite successful. Universal model programming complexes were constructed, two of which are described in [5] and the others are at the author's disposal. The interface part of such complexes allows to formulate problems by determining the model (differential or integral) of the researched process, the spatial-temporal domain of the functioning process and forming observing system (discrete or continuous) for its initial-boundary condition. Due to the lack of bench scale experiments for real observations, corresponding dynamic distributed process condition and the function of distributed external-dynamic perturbations were given. Accordingly, the points and spatial-temporal intervals of observations were defined. Despite the limited number of points observations and points, in which modeling functions, practical implementation of mathematical modeling algorithms solutions of the considered problems were agreed with the selected function of its spatial-temporal dynamics. The duration of problem solution didn't take much time (within academic hour).

Below, in addition to the considered results of the practical implementation of the proposed by us mathematical modeling algorithms in [5] of the incompletely observed spatially distributed systems is given the interface of a program-modeling complex for mathematical modeling dynamics of discretely observable spatially distributed processes in limited spatial-temporal domains with the use of discretely defined modeling functions and the result of its testing on a two-dimensional hyperbolic system, which in the laboratory works was described by students of Kyiv National Taras Shevchenko University in 2018.

The software implementation of the complex is written in Python programming language with the use of built-in math libraries. The interface is written in PyQt5, which is Python shell for the Qt library. By writing both parts of the complex in the same language, the information between them is transmitted faster, than in the case of using different environments for external shell and mathematical calculations.

Due to the limited visualization of 4-dimensional and higher spatial domains, it was decided to allow only one- and two- dimensional spatial domains. In this case required temporal variable is obligatory, which testifies to the dynamic peculiarity of the process.

The process of input data by the user is obviously clear from the interface; all calculations are performed internally and are protected from the user's impact, after which the result of modeling is displayed on the screen.

The program is launched by opening the tab «Model», where general process information is described.

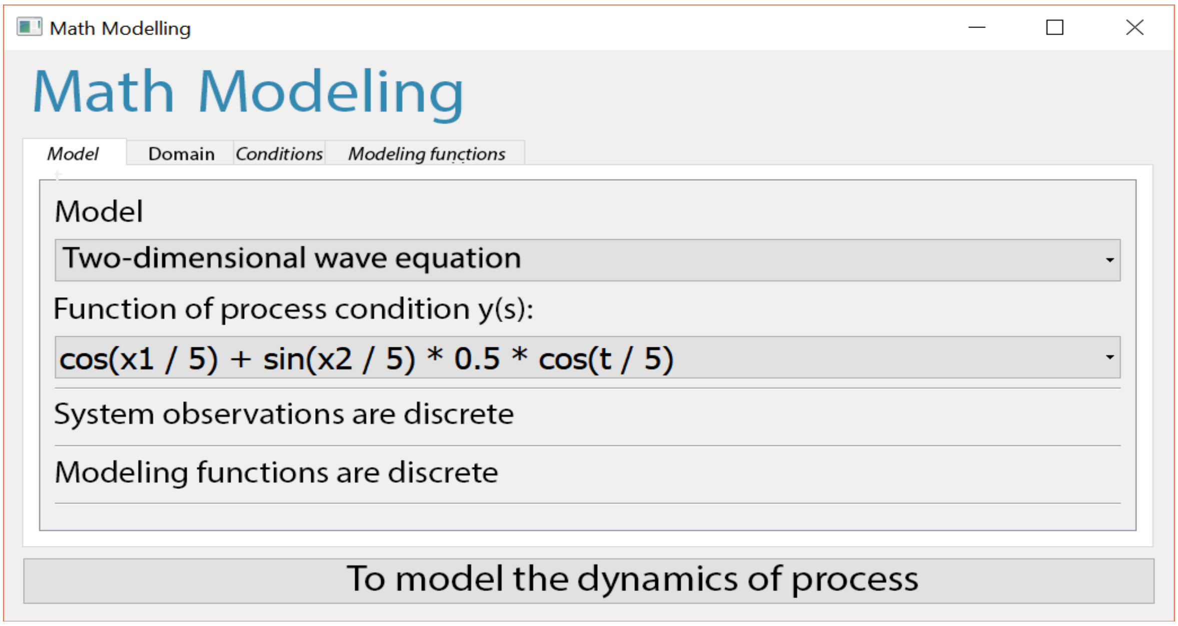

Fig. 1. The interface of input process description

The description of logic of the researched process is conducted by selecting the desired model from the list by name. In this case, in the program automaticallyare determined corresponding to it differential operator and Green's function$G\ (s - s^{'})$.

Depending on the number of spatial variables, the dimension of the problem is automatically determined. Dueto this, in the following tabs input data are immediately inserted for a set number of variables, which provides protection against input errors.

It is also possible to input (select from the list) of the exact solution $y(s)$of the problem. With its use, the function$u(s)$of distributed external-dynamic disturbances, which corresponds to it, is calculated. This provides protection against errors of user's input, when the functions are not consistent or the values at the selected for observations points aren't consistent with physics of process(values of functions are meant$Y_r^0(x)(r=\overline{1,R_0}; x \in S)$and $Y_p^{\Gamma}(x^{\Gamma},t)(p=\overline{1,R_{\Gamma}}; t \in [0,T])$in the selected points).

In the next tab – tab "Domain" geometry of spatial-temporal domain, in which the process is researched, is defined.

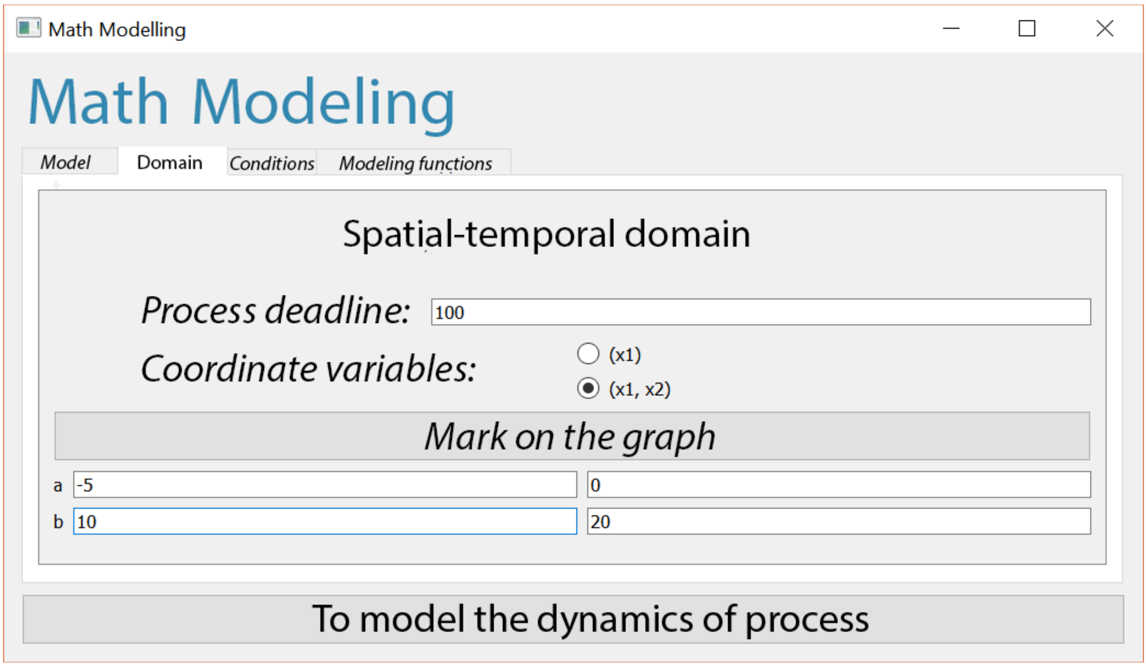

Fig. 2. The interface of input domain of processfunctioning.

The temporal interval begins with a zero-time stamp and is restricted by the value T which is introduced through the specially selected for this field.

Spatial coordinates are also restricted. If the domain is one-dimensional according to the spatial coordinates, then it is proposed to introduce points a and b, which correspond to boundaries for the given variable. Similarly, according to two spatial coordinates two corresponding boundaries are introduced – it means, that the spatial part of the domain is rectangular.

Initial and boundary observations for the process are introduced in the tab"Conditions".

The number of initial-boundary conditions can be arbitrary.Values $R_0$and R_{Γ} can immediately be written in the appropriate fields, and for the necessity to increase their number by using the corresponding buttons.

The initial conditions are determined under $t=0$in points$x \in S$.

The boundary conditions are defined in points$s_l^{\Gamma} \in \Gamma \times [0,T]$.

The number of spatial coordinates is determined with the use of information, previously introduced by the user.Due to these values$Y_r^0(x)(r=\overline{1,R_0}; x \in S)$and $Y_p^{\Gamma}(x^{\Gamma},t)(p=\overline{1,R_{\Gamma}}; t \in [0,T])$ are determined automatically from function introduced earlier, with added to it if desired noise function,if necessary.

After having finished of needed input data, it is necessary to click on the button "Save". At the same time the opportunity of changing the introduced conditions or add new or subtract the data, introduced earlier,remains.

Modeling functions are discrete and points, in which they are defined, are introduced in the tab "Modeling functions".

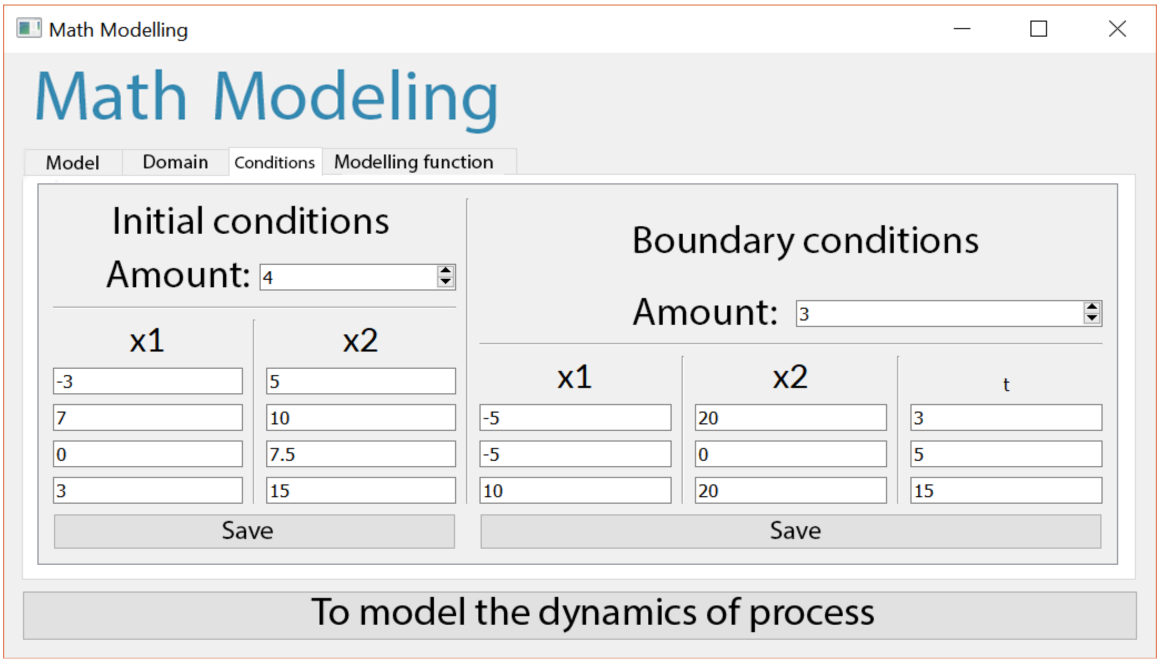

Fig. 3. The interface of input observations for the process

Points for modeling functions u^0, u^Γ should belong to the domains $S^0 = S \times (-\infty, 0]$ and$S^\Gamma = (R^n \backslash S) \times (0, T]$respectively.

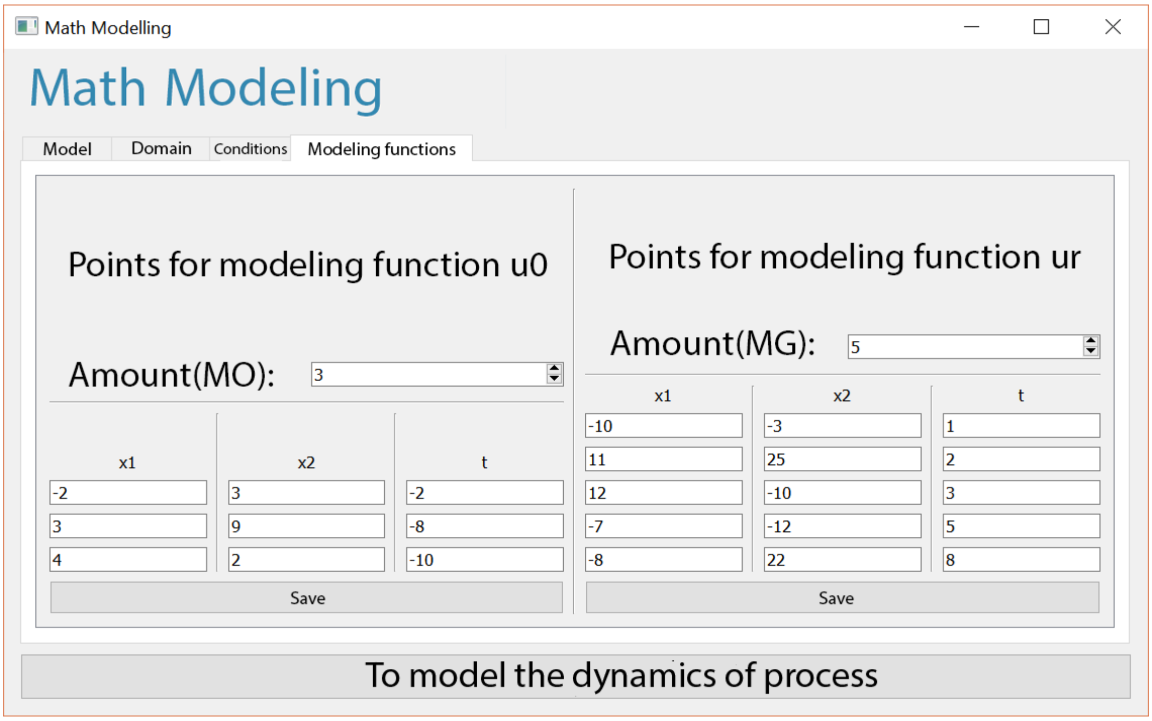

Fig. 4. The interface of input points of modeling functions points definition.

Inall the other cases logic of the interface follows the appropriate logic for introducing observations for the process.

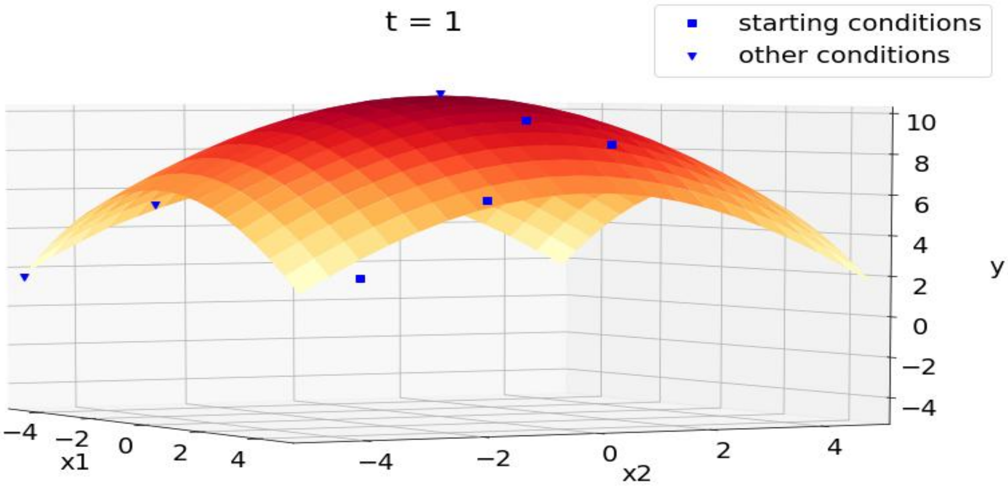

Fig. 5. The function of system condition under $t=1$ and initial-boundary observations for it.

When you click on the button "To model the dynamics of the process", begin calculations and, after which the new window graphic of functions $y(s)$ of system condition is displayed .The initial and boundary observations are also in the form of animation – for each$t \in [0,T]$.

Figure 5 shows the result of mathematical modeling of the considered process condition under consideration, which is observed through the condition $y(s) = 10\cos(0{,}2x_1) + \cos(0{,}2x_2) + sint$under consideration,that observation (in the figure they are shown by points) for its initial-boundary condition are conducted in points (1,1,0), (2,2, 0), (-4, 3, 0), (5, -4, 0) and (0, 0, 2), (0, -5, 3) (-5, -5, 7) respectively.The noise level of observations is 0.001.

II. CONCLUSIONS

Researching mathematical fundamentals of linear, dynamical, spatially distributed systems under conditions of not enough information about initial-boundary condition, are summarized and briefly presented. The cases are considered, when under linear, mathematically defined model of observation at the initial-boundary conditions are satisfied discretely and infinitely. The number and structure of such observations are knowhow, connected with the order and logic of the operator's differential models. There are no particular restrictions on size and structure of spatial-temporal domain, in which this process is researched. The system's function condition is constructed in such a way, that being model's differential solution according to the root mean-square criterion is agreed with supplied it observations. The evaluation of such level of agreement is given and the obtained solutions' unique conditions are formulated. A great attention in this research is devoted to issues of practical implementation of the suggested problem`s solutions.

Mentioned above and easily accessible in the research programs implementations allow simply and universally formulate the problem for any free linear dynamical system in any spatial-temporal domain at the presence of structurally infinite observations for systems condition. Algorithms' solution mathematical implementation of the considered problems' solution is simple and restricted by the use of the famous linear algebra methods of numerical and mathematical analyses.

Suggested methods' testing results are confirmed not only with mathematical problems' solution authentically and reliably, but also with access and simplicity of practical implementation.

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