

Steady State Response to Moving Line Load in Generalized Thermoelastic Half-Space

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Abstract: In this paper we have investigated the steady state response of an applied line load which is moving over a thermoelastic half space with uniform speed for a long time. The governing equations of generalized thermoelasticity on G-N model without dissipation of energy have been used to calculate the displacements, temperature and stresses by using Galilean transformation. Finally, solutions have been depicted by graphical presentation.

Keywords: Galilean transformation, generalized thermoelasticity, G-N model, half-space, steady state.

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I. Introduction

The classical theory of heat conduction predicts infinite speed of heat transportation when a homogeneous isotropic elastic solid is subjected to a thermal disturbance. This implies that the effect is felt at a location far from the source instantaneously. This paradox is due to the fact that in the classical theory, the equation of motion is hyperbolic in nature whereas the heat conduction equation used in the classical theory is parabolic. In order to avoid this discrepancy Lord and Shulman [1] introduced a flux-rate term in the Fourier law of heat conduction which results a finite speed of thermal signals. Green and Lindsay [2] included temperature rate dependent term in the constitutive equations that does not violate the classical Fourier law of heat conduction. This theory also predicts finite speed of heat propagation. According to these theories, heat propagation is to be viewed as a wave phenomenon rather than a diffusion phenomenon. Suhubi [3] referred to this wave like disturbances as “second sound”. In order to apply the heat flow theory in more wider class of heat flow problems, Green and Naghdi [4-6] made certain basic modification to this generalized theory. During the last four decades, a host of papers have been published following the models of these generalized theories. To mention a few of these one, two and three dimensional problems of generalized thermoelasticity with various thermal inputs the papers of Youseff and Bassiouny [7], Lotfy and Hassan [8], Kar and Kanoria [9], Banik and Kanoria [10], Sharma et al [11] may be named.

Moving load problems may be important in designing highways, airport runways or railway tracks etc. Sharma et al [12] aimed at investigating the steady state response of applied load moving with constant speed for an infinitely long time over the top surface of a homogeneous thermoelastic layer lying over a half space. Galilean transformation and Fourier transform technique have been applied to obtain the solution. Kumar and Rani [13] attempted the problem of deformation due to moving loads in thermoelastic body with voids. Sumi et al [14] used an analytical/numerical solution for transient stresses for a simplified 3-D problem. He described a local square surface heat source moving in the x-direction across an infinite flat plate in the xy-plane. Sumi and Hetnarski [15] considered a similar problem where the heat source is moving back and forth with a constant angular frequency over an infinite plate. For both the problems [14,15], solution is given for the transient thermal stresses for which transient temperature distributions are obtained by using Laplace and Fourier transforms. The associated stresses are obtained by making use of thermoelastic displacement function and the Galerkin function. For other important work in this area, the papers of Brock and Rodgers [16], Chakravorty and Chakravorty [17] may be mentioned.

As in Sharma et al [12], the present study is on the problem of steady state response to moving load in generalized thermoelastic half-space. Galilean transformation followed by complex variable method has been applied to solve the problem.

II. Basic Equations

Following Green and Naghdi (G-N) model without energy dissipation [5] the stress-strain-temperature relations, equations of motion and heat conduction equation for a homogeneous, isotropic thermoelastic body, in the absence of body forces and heat sources are given by

$$\tau_{ij} = \lambda(\vec{\nabla} \cdot \vec{u})\delta_{ij} + 2\mu e_{ij} - \gamma\theta\delta_{ij} \quad (1)$$

$$\mu\nabla^2\vec{u} + (\lambda + \mu)\vec{\nabla}(\vec{\nabla} \cdot \vec{u}) - \gamma\vec{\nabla}\theta = \rho\ddot{\vec{u}} \quad (2)$$

$$\rho c\ddot{\theta} + \gamma\theta_0\vec{\nabla} \cdot \ddot{\vec{u}} = k^*\nabla^2\theta \quad (3)$$

where $\theta(X, Y, t)$ is the change of temperature over θ_0 , $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermalexpansion; λ, μ are Lamé constants, ρ is the density, c is the specific heat at constant strain, k^* is the characteristic constant of the medium. The heat transport equation (3) predicts the finite speed of $\left(\frac{k^*}{\rho c}\right)^{\frac{1}{2}}$ for thermal signal.

III. Formulation of the Problem

Let the origin of the rectangular Cartesian coordinate system OXYZ be fixed at any point on the boundary of the homogeneous, isotropic, thermally conducting half-space with Y-axis vertically downwards and the Z-axis as shown in the figure.

A normal line load (occurring parallel to the direction of Y-axis) moves with uniform velocity U in the negative direction of the X-axis. It is assumed that the load has been moving steadily for a long time so that a steady stress pattern appears to an observer moving with the load. Therefore, all the quantities are independent of the Z-coordinate and the Z-component of velocity also vanishes everywhere.

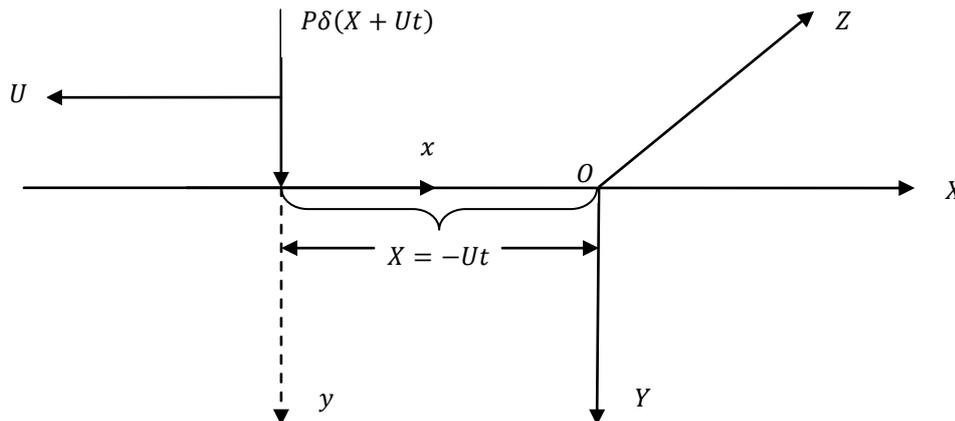


Figure1. Load distribution

In the state of plane strain, we consider the displacement of the form $\vec{u} = (u, v, 0)$.

We now introduce two potential functions ϕ and $\vec{\psi}$ such that

$$\vec{u} = \text{grad } \phi + \text{curl } \vec{\psi}, \quad \text{div } \vec{\psi} = 0 \quad (4)$$

The components of displacement can be written as

$$u = \frac{\partial\phi}{\partial X} - \frac{\partial\psi}{\partial Y}, \quad v = \frac{\partial\phi}{\partial Y} + \frac{\partial\psi}{\partial X}$$

where $\vec{\psi} = (0, 0, -\psi)$.

Substituting (4) in (2), we obtain

$$\vec{\nabla}[(\lambda + 2\mu)\nabla^2\phi - \gamma\theta - \rho\ddot{\phi}] + \vec{\nabla} \times [\mu\nabla^2\vec{\psi} - \rho\ddot{\vec{\psi}}] = \vec{0}$$

This equation will be satisfied if each of the bracketed quantity vanishes. Thus we get

$$c_1^2\nabla^2\phi - \frac{\gamma}{\rho}\theta = \ddot{\phi} \tag{5}$$

$$c_2^2\nabla^2\psi = \ddot{\psi} \tag{6}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}$$

Using (4) equation (3) becomes

$$\rho c \frac{\partial^2\theta}{\partial t^2} + \gamma\theta_0\nabla^2\ddot{\phi} = k^*\nabla^2\theta \tag{7}$$

Eliminating θ between (5) and (7) we get

$$\frac{k^*\rho c_1^2}{\gamma}\nabla^4\phi - \left(\frac{k^*\rho}{\gamma} + \gamma\theta_0 + \frac{\rho^2 c c_1^2}{\gamma}\right)\nabla^2\left(\frac{\partial^2\phi}{\partial t^2}\right) + \frac{\rho^2 c}{\gamma}\frac{\partial^4\phi}{\partial t^4} = 0 \tag{8}$$

IV. Boundary Conditions

In the problem the normal line load moves with uniform velocity U in the negative X – direction. So the mechanical and thermal boundary conditions for this concentrated moving line load on the surface are

$$\left. \begin{aligned} \text{(i)} \quad & \tau_{YY} = -P\delta(X + Ut), \quad \tau_{XY} = 0 \\ \text{(ii)} \quad & \Theta = 0 \end{aligned} \right\} \text{ on } Y = 0 \tag{9}$$

Since an observer moving with the load of uniform velocity U would see the load as stationary, we may assume that X and t enter in ϕ and ψ in combination of $X + Ut$ only.

Hence, we introduce the Galilean transformation

$$x = X + Ut, \quad y = Y \tag{10}$$

Using (10), all the above equations as well as ϕ and ψ will be functions of x and y only.

Thus equations (5), (6) and (7) become, respectively

$$\beta_1^2 \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \frac{\gamma\theta}{\rho c_1^2} \tag{11}$$

$$\beta_2^2 \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \tag{12}$$

$$\rho c U^2 \frac{\partial^2\theta}{\partial x^2} + \gamma\theta_0 U^2 \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} \right) = k^* \left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} \right) \tag{13}$$

where

$$\beta_1^2 = 1 - \frac{U^2}{c_1^2} = 1 - M_1^2, \quad \beta_2^2 = 1 - \frac{U^2}{c_2^2} = 1 - M_2^2 \text{ and } M_1 = \frac{U}{c_1}, \quad M_2 = \frac{U}{c_2} \tag{14}$$

Eliminating θ between (11) and (13), we get

$$P_1 \frac{\partial^4\phi}{\partial x^4} + P_2 \frac{\partial^4\phi}{\partial x^2\partial y^2} + P_3 \frac{\partial^4\phi}{\partial y^4} = 0 \tag{15}$$

where

$$P_1 = \frac{\rho c_1^2}{\gamma} (k^* - \rho c U^2) \beta_1^2 - \gamma\theta_0 U^2$$

$$P_2 = \frac{\rho c_1^2}{\gamma} \{k^*(1 + \beta_1^2) - \rho c U^2\} - \gamma \theta_0 U^2$$

$$P_3 = \frac{k^* \rho c_1^2}{\gamma}$$

The components of displacements and stresses can be calculated as

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}$$

$$\frac{\tau_{xx}}{\mu} = M_2^2 \frac{\partial^2 \phi}{\partial x^2} - 2 \frac{\partial^2 \phi}{\partial y^2} - 2 \frac{\partial^2 \psi}{\partial x \partial y}$$

$$\frac{\tau_{xy}}{\mu} = 2 \frac{\partial^2 \phi}{\partial x \partial y} + (2 - M_2^2) \frac{\partial^2 \psi}{\partial x^2}$$

$$\frac{\tau_{yy}}{\mu} = (M_2^2 - 2) \frac{\partial^2 \phi}{\partial x^2} + 2 \frac{\partial^2 \psi}{\partial x \partial y}$$

From (9), the mechanical stress boundary conditions for the moving coordinate system on the surface $Y = 0$ when $x = X + Ut, y = Y$ can be written as

$$\tau_{yy} = -P\delta(x), \quad \tau_{xy} = 0 \text{ on } y = 0 \tag{16}$$

So, the boundary conditions (16) at $y = 0$ become

$$-(2 - M_2^2) \frac{\partial^2 \phi}{\partial x^2} + 2 \frac{\partial^2 \psi}{\partial x \partial y} = -\frac{P}{\mu} \delta(x) = -\frac{P}{\mu} H'(x) \text{ at } y = 0 \tag{17}$$

$$2 \frac{\partial^2 \phi}{\partial x \partial y} + (2 - M_2^2) \frac{\partial^2 \psi}{\partial x^2} = 0 \text{ at } y = 0 \tag{18}$$

Assuming the boundary as isothermal, we write the thermal boundary condition from equation (11) as

$$\theta = \frac{\rho}{\gamma} (c_1^2 - U^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\rho c_1^2}{\gamma} \frac{\partial^2 \phi}{\partial y^2} = 0 \text{ on } y = 0 \tag{19}$$

Integrating (17) and (18) with respect to x , we get at $y = 0$

$$-(2 - M_2^2) \frac{\partial \phi}{\partial x} + 2 \frac{\partial \psi}{\partial y} = -\frac{P}{\mu} H(x) \text{ at } y = 0 \tag{20}$$

$$2 \frac{\partial \phi}{\partial y} + (2 - M_2^2) \frac{\partial \psi}{\partial x} = 0 \text{ at } y = 0 \tag{21}$$

Thus the solution of our problem is given by θ, ψ and ϕ which satisfy the equations (11) – (13) along with the boundary conditions (19) – (21).

V. Solution of the Problem

For both $M_1, M_2 < 1$, we have from (14) β_1^2 and β_2^2 are positive. In this case the potential functions ψ and ϕ in (12) and (15) are satisfied by the real parts of any arbitrary analytic functions f and g of complex variables $(x + i\alpha y)$ and $(x + i\beta_2 y)$ respectively. Thus we write

$$\phi = \text{Re } f(x + i\alpha y) \tag{22}$$

$$\psi = \text{Re } g(x + i\beta_2 y) \tag{23}$$

Equation (23) satisfies the equation (12) exactly. Now substituting (22) in (15) we get

$$P_3 \alpha^4 - P_2 \alpha^2 + P_1 = 0$$

$$\Rightarrow \alpha = \left[\frac{P_2 \pm (P_2^2 - 4P_1P_3)^{\frac{1}{2}}}{2P_3} \right]^{\frac{1}{2}} \tag{24}$$

where α must be real.

The thermal boundary condition (19) yields

$$\alpha^2 = 1 - \frac{U^2}{c_1^2}$$

$$\Rightarrow U < c_1 \tag{25}$$

since α is real.

The mechanical boundary conditions (20) and (21) give

$$(2 - M_2^2) \operatorname{Re} f'(x + i\alpha y) - 2 \operatorname{Re} i \beta_2 g'(x + i\beta_2 y) = \frac{P}{\mu} H(x)$$

and $\operatorname{Re}[2i\alpha f'(x + i\alpha y) + (2 - M_2^2) g'(x + i\beta_2 y)] = 0$

Putting $y = 0$ in the above two equations, we get

$$\left. \begin{aligned} \operatorname{Re} g'(x) &= -\operatorname{Re} \frac{2i\alpha}{2 - M_2^2} f'(x) \\ \operatorname{Re} f'(x) &= \frac{K_1 P}{\mu} H(x) \end{aligned} \right\} \quad (26)$$

where

$$K_1 = \frac{2 - M_2^2}{(2 - M_2^2)^2 - 4\alpha\beta_2}$$

Now f' is a function of $x + i\alpha y = r_1 e^{i\theta_1}$ (say) where $r_1^2 = x^2 + \alpha^2 y^2$, $\theta_1 = \tan^{-1} \frac{\alpha y}{x}$

From (26) we get

$$\left. \begin{aligned} \text{(i) At } \theta_1 = 0, x > 0, \operatorname{Re} f' &= \frac{K_1 P}{\mu} \\ \text{(ii) At } \theta_1 = \pi, x < 0, \operatorname{Re} f' &= 0 \end{aligned} \right\} \quad (27)$$

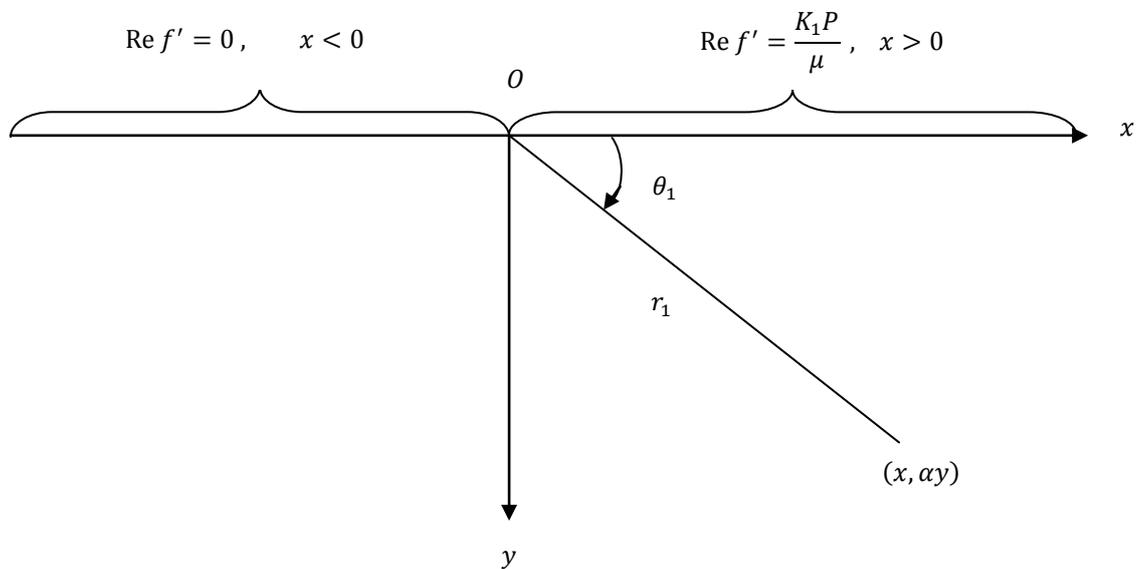


Figure 2. Geometry in Polar Coordinates

We note that

$$\begin{aligned} \operatorname{Re} \left[\frac{i}{\pi} \log(x + i\alpha y) \right] &= \operatorname{Re} \left[\frac{i}{\pi} \log(r_1 e^{i\theta_1}) \right] \\ &= -\frac{\theta_1}{\pi} = \begin{cases} 0 & \text{when } \theta_1 = 0 \\ -1 & \text{when } \theta_1 = \pi \end{cases} \end{aligned}$$

$$\therefore \operatorname{Re} \frac{K_1 P}{\mu} \left[\frac{i}{\pi} \log(x + i\alpha y) + 1 \right] = \begin{cases} \frac{K_1 P}{\mu} & \text{when } \theta_1 = 0 \\ 0 & \text{when } \theta_1 = \pi \end{cases}$$

Hence, we take

$$f'(x + i\alpha y) = \frac{K_1 P}{\mu} \left[\frac{i}{\pi} \log(x + i\alpha y) + 1 \right]$$

$$\therefore f'(x) = \frac{K_1 P}{\mu} \left(\frac{i}{\pi} \log x + 1 \right) \tag{28}$$

By the help of the first equation of (26) and (28) we write

$$\text{Re } g'(x) = -\text{Re} \frac{2i\alpha}{(2 - M_2^2)} \frac{K_1 P}{\mu} \left(\frac{i}{\pi} \log x + 1 \right)$$

Thus, we get

$$g'(x) = \frac{2\alpha K_1 P}{\mu(2 - M_2^2)} \left(\frac{1}{\pi} \log x - i \right)$$

$$\therefore g'(x + i\beta_2 y) = \frac{K_2 P}{\mu} \left[\frac{1}{\pi} \log(x + i\beta_2 y) - i \right] \tag{29}$$

where

$$K_2 = \frac{2\alpha}{(2 - M_2^2)^2 - 4\alpha\beta_2}$$

Let $x + i\beta_2 y = r_2 e^{i\theta_2}$ (say) where $r_2^2 = x^2 + \beta_2^2 y^2$, $\theta_2 = \tan^{-1} \frac{\beta_2 y}{x}$

Using (15), (22) and (23), the components of displacements and stresses can be calculated as

$$u = \text{Re}[f'(x + i\alpha y)] - \text{Re}[i\beta_2 g'(x + i\beta_2 y)]$$

$$= \frac{K_1 P}{\mu} \left(1 - \frac{\theta_1}{\pi} \right) - \frac{K_2 P}{\mu} \beta_2 \left(1 - \frac{\theta_2}{\pi} \right) \tag{30}$$

$$v = \text{Re}[i\alpha f'(x + i\alpha y) + g'(x + i\beta_2 y)]$$

$$= \frac{P}{\pi\mu} (K_2 \log r_2 - K_1 \beta_1 \log r_1) \tag{31}$$

Similarly,

$$\pi\tau_{xx} = K_1 P (M_2^2 - 2M_1^2 + 2) \frac{\sin \theta_1}{r_1} - 2K_2 P \beta_2 \left(\frac{\sin \theta_2}{r_2} \right) \tag{32}$$

$$\pi\tau_{yy} = K_1 P (M_2^2 - 2) \frac{\sin \theta_1}{r_1} + 2K_2 P \beta_2 \left(\frac{\sin \theta_2}{r_2} \right) \tag{33}$$

$$\pi\tau_{xy} = 2K_1 P \alpha \left(\frac{\cos \theta_2}{r_2} - \frac{\cos \theta_1}{r_1} \right) \tag{34}$$

Using (11), the temperature θ takes the form

$$\theta = \frac{\rho c_1^2 K_1 P}{\pi\mu\gamma} (\beta_1^2 - \alpha^2) \left(\frac{\sin \theta_1}{r_1} \right) \tag{35}$$

VI. Numerical Results

To illustrate the problem graphically, we take the case of magnesium crystal-like material for numerical evaluation.

Following Dhaliwal and Singh [18], the physical constants are taken as

$$\lambda = 2.17 \times 10^{10} Nm^{-2}, \quad \mu = 3.278 \times 10^{10} N m^{-2}, \quad \Theta_0 = 298^\circ K,$$

$$\rho = 1.74 \times 10^3 Kgm^{-3}, \quad k^* = 1.7 \times 10^2 Wm^{-1} degree^{-1},$$

$$c = 1.04 \times 10^3 J Kg^{-1} degree^{-1}, \quad P = 1$$

Figure 3 – Figure 6 represent the variations of temperature, normal stresses τ_{xx}, τ_{yy} and shear stress τ_{xy} with distance x along the line $y = mx$, where $m = \tan \theta$, for different values of the angle θ .

Figure 3 depicts the variation of temperature Θ along x for $\theta = 0^\circ, 30^\circ, 45^\circ$. It is observed that at $x = 0$ the temperature θ assumes positive values of 6.133×10^{-7} and 7.135×10^{-7} for $\theta = 30^\circ, 45^\circ$ respectively. Then it gradually decreases for each case.

Figure 4 represents the variation of normal stress τ_{xx} along the distance x for different values of θ . For GN-II theory, τ_{xx} attains the positive values of 2.558 and 3.496 for the acute angles $\theta = 30^\circ, 45^\circ$ at $x = 0$ and then gradually decreases whereas for CTE theory it attains negative values of -1.413 and -1.094 for $\theta = 30^\circ, 45^\circ$ at $x = 0$ and then gradually increases.

Figure 5 represents the variation of normal stress τ_{yy} along the distance x . For both the theories (GN-II and CTE) the values of τ_{yy} increases as the angle of inclination decreases from $\theta = 45^\circ$ to $\theta = 0^\circ$. For each GN-II and CTE theory, τ_{yy} assumes minimum values of $-0.4592, -1.054$ and $-0.4488, -1.06$ at $x = 0$ for $\theta = 30^\circ, 45^\circ$ respectively. Then for both the cases τ_{yy} tends to zero as x increases.

Figure 6 predicts the variation of shear stress τ_{xy} along the distance x for different values of $\theta = 0^\circ, 30^\circ$ and 45° . For GN-II model, τ_{xy} attains the minimum values of -0.7747 and -1.046 whereas for CTE theory it attains minimum values of -0.8101 and -1.087 at $x = 0$ for $\theta = 30^\circ, 45^\circ$ respectively. Then for both the cases τ_{xy} gradually increases.

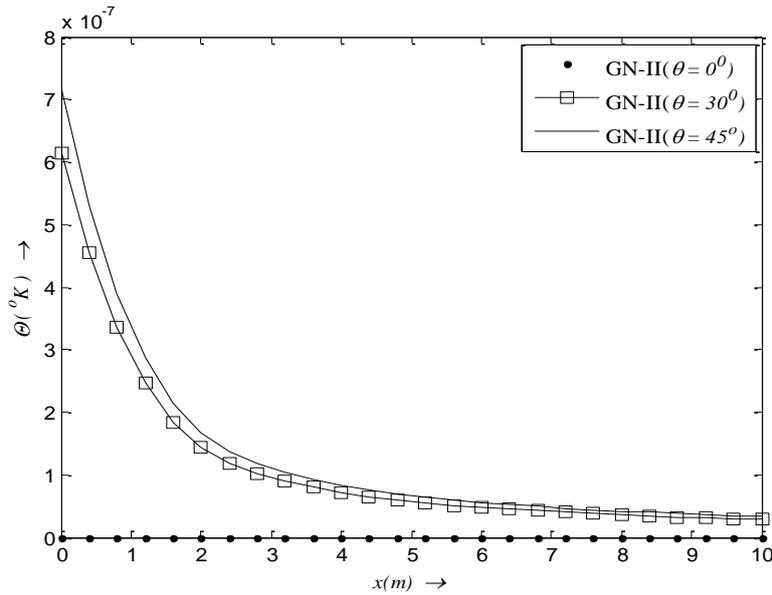


Figure 3. Variation of temperature with distance x

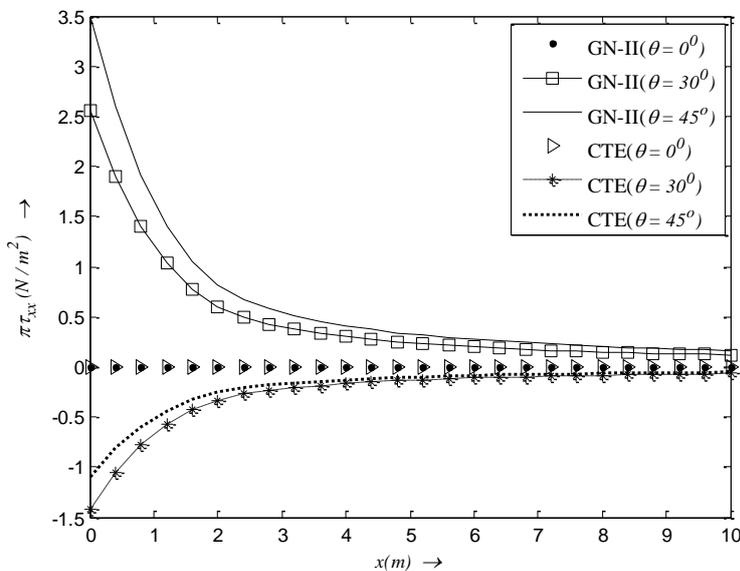


Figure 4. Variation of normal stress τ_{xx} with distance x

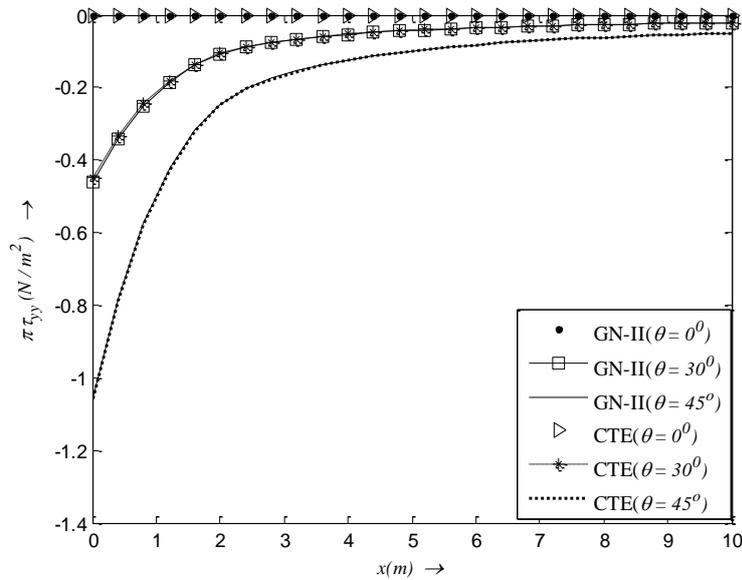


Figure 5. Variation of normal stress τ_{yy} with distance x

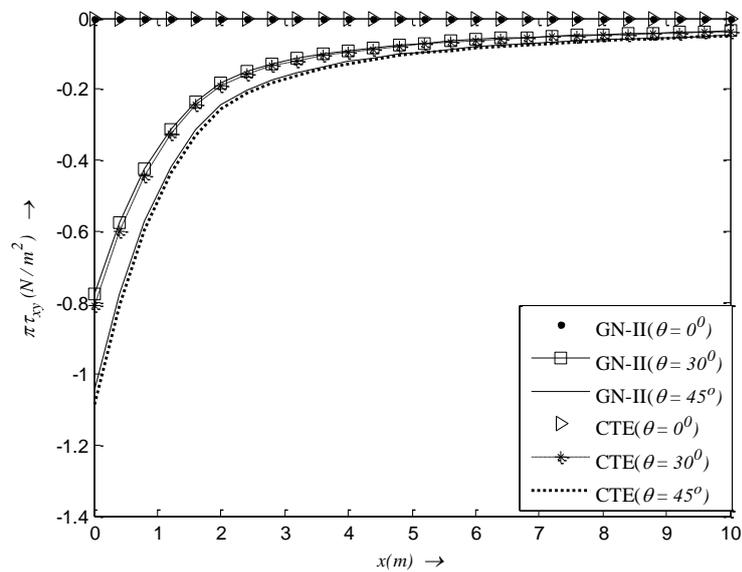


Figure 6. Variation of shear stress τ_{xy} with distance x

VII. Conclusion

Thus, we may infer that when θ_1 increases from its initial position, then at a particular value of the angle, the comparison in values of the characteristic changes of the field variables of our problem in different planes can be estimated. Similar conclusions can be drawn in respect of the angle θ_2 .

It may be noticed that the graphs of temperature and normal stresses as in Figures [3-5] for different cases in $0 < \theta < \frac{\pi}{2}$ are symmetrical about the x-axis to the corresponding cases when $\frac{\pi}{2} < \theta < \pi$ is considered.

From the above discussion it is further noticed that the variations of normal stress τ_{yy} and shear stress τ_{xy} with distance x for different angles of inclination are similar in nature for Generalized Thermoelasticity(GN-II) and Classical Thermoelasticity(CTE) with difference in numerical values.

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