

# Undetermined Mixing Matrix Estimation Base on Classification and Counting

Xie Qiongyong, Zhang Wei

College Of Information Science and Technology, Jinan University, China

---

**ABSTRACT:** This paper introduces the mixing matrix estimation algorithms about undetermined blind source separation. Contrapose the difficulty to determine the parameters, the complex calculations in potential function method and the difficulty to confirm the cluster centers in method of clustering, we propose a new method to estimate the mixing matrix base on classification and counting the observed data. The experiment result shows that the new algorithm can not only simplify the calculation but also is easier to be understood. Besides, the new algorithm can provide a more accurate result according to the precision people wanted.

**Keywords:** undetermined blind source separation; potential function method; statistical analysis; sparse component analysis;

---

## I. INTRODUCTION

The Blind Source Separation is an approach to estimate the underlying sources from multiple observations(mixtures) received by a set of sensors.Recently, blind source separation technology has been widely applied in speech signal processing, image processing, radar, communication and other fields. When the number of observed signals less than the sources, we call it as Underdetermined Blind Source Separation (UBSS).

To solve the Underdetermined Blind Source Separation problems, we usually use Sparse Component Analysis (SCA). Firstly, use potential or clustering functions to estimate the mixing matrix, then recover source signals through the shortest path method. Bofill invented the shortest path method based on  $l_1$  norm criterion, and This invented the super complete linear ICA geometry algorithm. Document [2] presented recovering source signals based on pseudo extract vector, which improves the speed of signal separation. This paper proposes that split the axis and then calculate the observed data of each group to estimate the mixing matrix. This method can obtain a higher estimation accuracy by changing the number of group and shorten the time of running.

### 1.UBSS Model

For the UBSS, model can be written as

$$X(t) = \begin{bmatrix} x_1(t) \\ x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m3} & \cdots & a_{mn} \end{bmatrix} = AS(t)$$

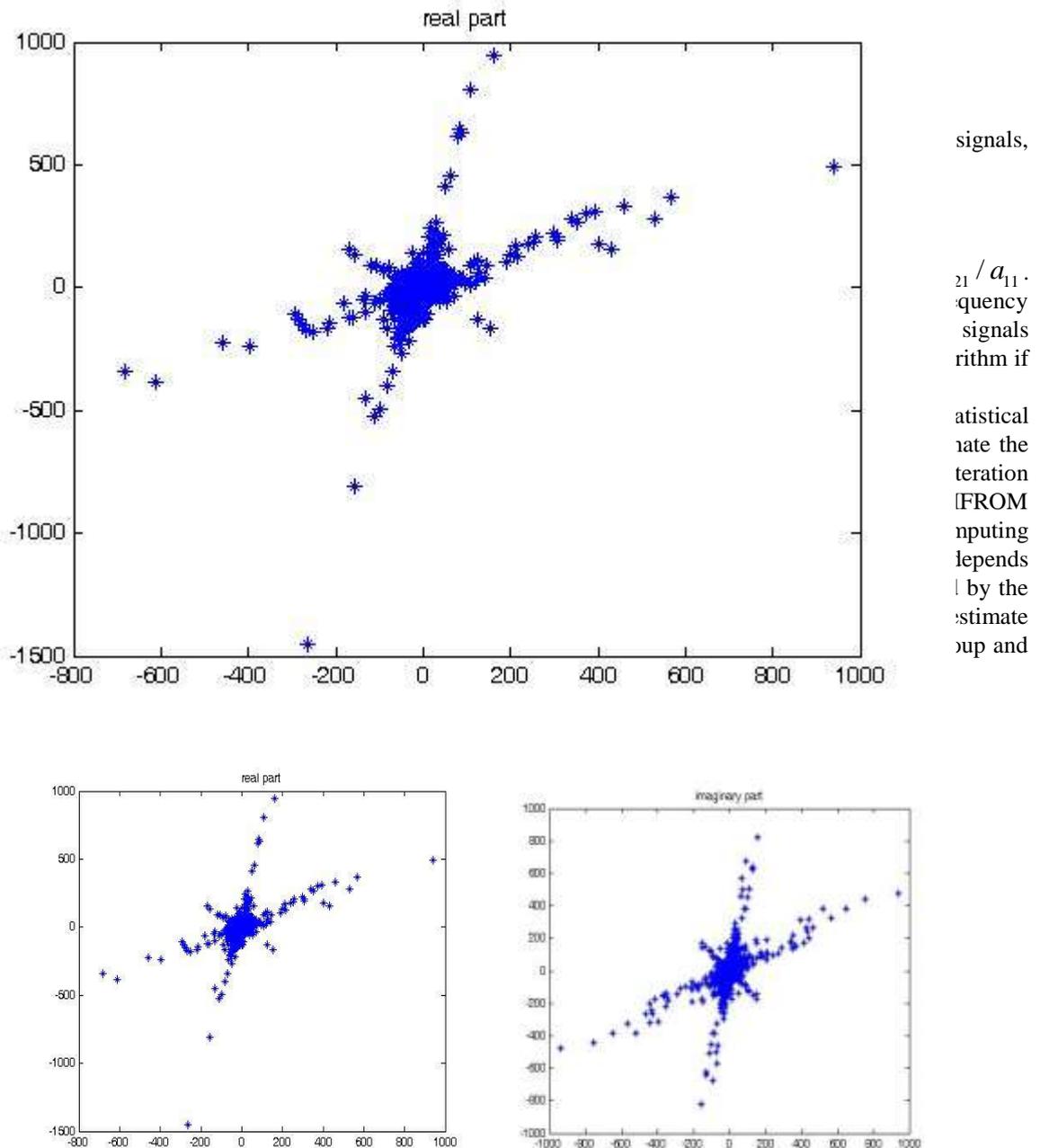
Where  $X(t)$  represents the  $M$ -dimensional observations,  $A$  means  $m \times n$  dimensional unknown mixing matrix,  $S(t)$  represents the underlying  $N$ -dimensional sources which are assumed to be mutually uncorrelated and  $T$  means the observation moment.

To solve the UBSS problems, some constrains must be put forward, the source signals are sparse signals which means there are two different dimensions have nonzero correlations. Assumed at sampling time  $t$ , only  $s_1(t)$  has nonzero values, the model can be written as

$$\begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix} = \begin{bmatrix} a_{11} \\ \vdots \\ a_{m1} \end{bmatrix} s_1(t) \Rightarrow \frac{x_1(t)}{a_{11}} = \frac{x_2(t)}{a_{21}} = \cdots = \frac{x_m(t)}{a_{m1}} = s_1(t)$$

**2. The estimation of mixing matrix**

Usually, we use potential function or clustering method to estimate the mixing matrix. Assuming that there are 3 sparse source signals ( $n=3$ )  $s_1, s_2, s_3, \dots$ , after mixing by a  $2 \times 3$  matrix, we can get two mixing observation signals ( $m=2$ ), written as follows



**Figure 1** the scatter diagram of 3 voice signals

The introduction of new algorithm:

Known from the analysis of the above, the sparse signal in sparse domain would gather in a straight line with the slope of the mixing matrix column vector. We can only focus on the first and the fourth quadrant because of the linear symmetrical about the origin. Dividing the first and the fourth quadrant into  $N$  groups, the slope range will be  $[-\pi/2, \pi/2]$  and the class interval is  $\pi/N$ , then calculate the angle of each observed point  $\theta = \tan^{-1}(\frac{y_{21}}{x_{21}})$  and count the number of points scattered in each group, the slope of clustering lines is in the groups where the peaks occur so that we can have the mixing matrix, the algorithm steps are as follows

1) split the first and the fourth quadrant into N groups, the group interval is  $\pi/N$ , slope range of each group is  $[\theta_i, \theta_i + \pi / N]$

2) calculate the angle of each observation point  $\theta = \tan^{-1}(\frac{x_{21}}{x_{11}})$ , move the point to the corresponding group according to the angle.

3) we can get the number of the straight line n from the scatter diagram so we consider the n groups as the slope of the straight lines who has the most observation points. If there are some continuous peaks among these groups, then we need to subdivide those continuous groups, repeat step 2) and 3) until all the groups have obvious single peak.

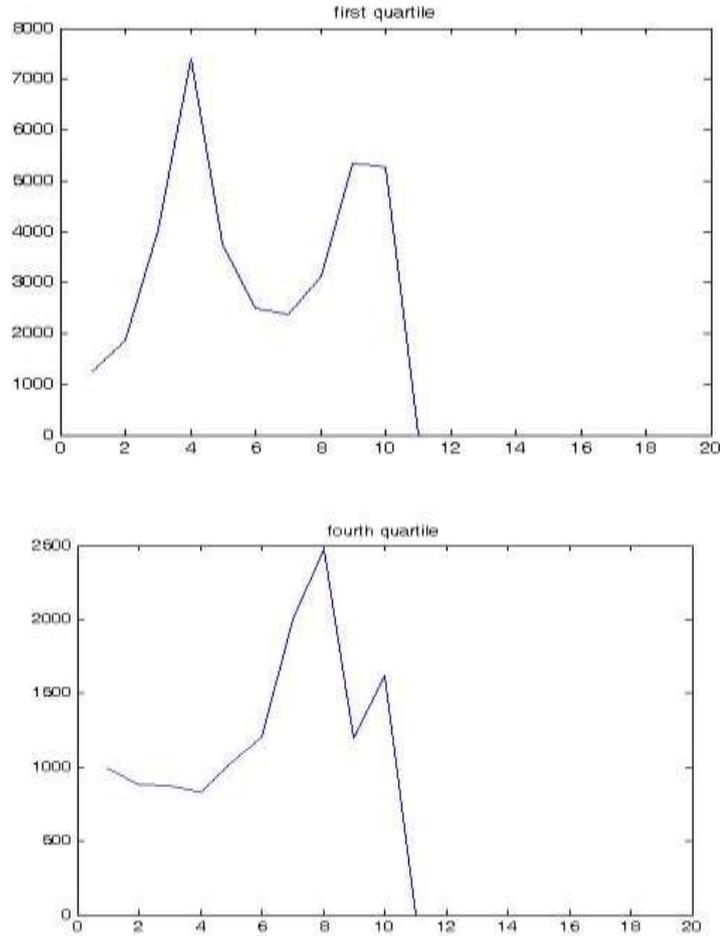
4) Consider the group of every single peak as the slope of the clustered straight lines, repeat step 3) if we need to make more precise result.

## II. SIMULATION

Choose 3 voice signals, the mixing matrix is

$$A = \begin{bmatrix} \cos \frac{\pi}{6} & \cos \frac{4\pi}{9} & \cos \frac{3\pi}{4} \\ \sin \frac{\pi}{6} & \sin \frac{4\pi}{9} & \sin \frac{3\pi}{4} \end{bmatrix}$$

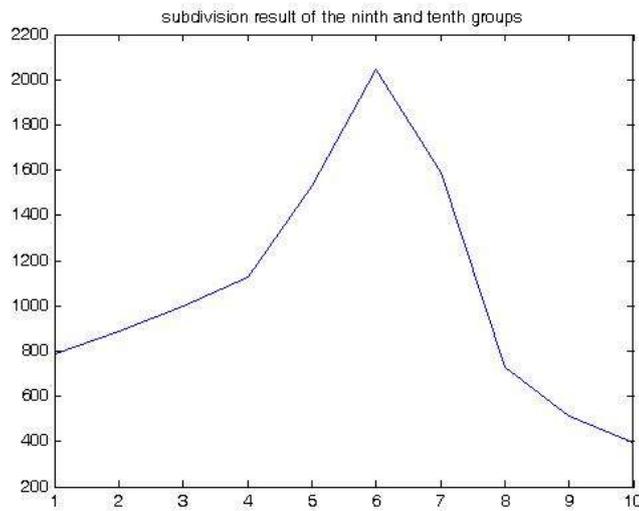
According to the estimating mixing matrix method of this paper, divide the first and the fourth quadrant into 20 groups (N=20), the number of observation points of each group has showed in figure 2.



**Figure 2** the first and fourth quadrant result

From the scatter diagram we can see that the first quadrant has 2 straight lines, and the peaks occur in the fourth, ninth and the tenth group in figure2, so one slope of the straight line in the first quadrant should match with the fourth group (among  $[\frac{3\pi}{20}, \frac{4\pi}{20}]$ ), then subdivide the ninth and the tenth group, the statistical results are shown in figure 3 from where we can see the single peak is in the six group so that we can infer the range of this peak is  $[\frac{45\pi}{100}, \frac{46\pi}{100}]$  because of the subdivide range is  $[\frac{8\pi}{20}, \frac{10\pi}{20}]$ .

In the same way, we can see that there is one straight line in the scatter diagram, so we choose the group which has the largest number of observation points, the fourth group (among  $[-\frac{3\pi}{20}, -\frac{4\pi}{20}]$ ) as the slope of the straight line in the fourth quadrant.



**Figure 3** the subdivision result

From what has been discussed above, the slope ranges of 3 straight lines should be  $[-\frac{3\pi}{20}, -\frac{4\pi}{20}]$ ,  $[\frac{7\pi}{20}, \frac{8\pi}{20}]$  and  $[\frac{45\pi}{100}, \frac{46\pi}{100}]$ . As for the specific slope, we can use the interval midpoint as the approximate

value. If it is required to meet the accuracy of  $\frac{\pi}{100}$ , we can subdivide these intervals and reach the specific

slope as  $\frac{33\pi}{200}, \frac{9\pi}{20}, -\frac{9\pi}{50}$ , the mixing matrix will be

$$\tilde{A} = \begin{bmatrix} \cos \frac{33\pi}{200} & \cos \frac{9\pi}{20} & \cos \frac{9\pi}{50} \\ \sin \frac{33\pi}{200} & \sin \frac{9\pi}{20} & \sin \frac{9\pi}{50} \end{bmatrix}$$

The error matrix between estimation matrix and the real mixing matrix is

$$\tilde{A} - A = \begin{bmatrix} 0.002 & -0.017 & 0.13 \\ -0.004 & 0.002 & -0.17 \end{bmatrix}$$

From the experiment above we can see that the error between the estimation and real mixing matrix is

quite small, besides, the potential function and its window function is

$$\phi(\theta) = \sum_i l_i^2 \varphi(\eta(\theta_i - \theta))$$

$$\varphi(\alpha) = \exp(-|\partial|) \quad |\alpha| < \frac{\pi}{4}$$

$$\phi(\theta) = \sum_i l_i^2 \varphi(\theta)$$

$$\varphi(\alpha) = \exp(-|\partial|) \quad |\alpha| < \frac{\pi}{4}$$

The computing complexity of the potential function method will be index computing level, which needs to spend more time during the operation. However, the new method in this paper, we just classify the angle of each observation point, the time complexity is O(n), so the new method is simple and spend less time.

### III. CONCLUSION

This paper introduces potential and the clustering method in estimating the mixing matrix, proposes a new method based on the shortcoming of these 2 methods above. The new method subdivide the groups according to requirement of accuracy. The new method is more simple and needs less time for operation compared to the potential function and the clustered method.

### REFERENCES

- [1]. Comon P, Jutten C. Handbook of blind source separation: independent component analysis and application [M] Kidlington, Oxford, UK: Academic Press of Elsevier, 2010
- [2]. Bai Lin, New method of underdetermined blind voice source separation [J] Application Research of Computers 2010 27(7)
- [3]. Chen Xiaojun, Zhang Yang, Tang Bin, Undertetermined Blind Separation Based on Ratio Matrix Clustering [J] The modern electronic technology 2008 31(19):1-3
- [4]. Bofill P and Zibulevsky M. Undetermined blind source separation using sparse representations [J] Signal Processing, 2001, S1(11):2353-2362
- [5]. Theis Fj, Georgiev P, Cichocki A. Robust sparse component analysis based on a generalized Hough transform [J] EURASIP Journal on Advances in Signal Processing, 2006, 2007
- [6]. Fu Yongqing, Guo Hui, Unconstrained underdetermined blind source separation algorithm based on the source number estimation [J] Journal of Harbin Engineering University
- [7]. Y. Li, A. Cichocki, S Amari, Analysis of Sparse Representation and Blind Source Separation, Neural Computation, 2004, 16, 1193-1234.
- [8]. Qiu Tianshuang, Bi Xiaohui, Sparse Component Analysis and Application for Underdetermined Blind Source Separation [J] Signal Processing
- [9]. Balan R, Rosca J. Source separation using sparse discrete prior models. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2006, 4: 1113-1116
- [10]. Lv Q, Zhang X D. A unified method for blind separation of sparse sources with unknown source number. IEEE Signal Proc. Letters, 2006, 13(1): 49-51
- [11]. Pierre C. Blind identification and source separation in 2 x 3 underdetermined mixtures. IEEE Trans. on Signal Process. 2004, 52(1): 11-22
- [12]. M. Zibulevsky, B. A. Pearlmutter, Blind source separation by sparse decomposition in a signal dictionary [J], Neural Comput. 13, 2001, 863-882
- [13]. Barry G. Quinn Estimation frequency by interpolation using Fourier coefficients [J] IEEE Transactions on Signal Processing, 1994, Vol. 4
- [14].