

Holographic Computation of Three-dimensional Objects

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ABSTRACT: According to a series of theoretical knowledge of computational holography, First of all, two basic kinds of computational holographic methods were introduced. The first was to use three-dimensional scanner and three-dimensional software to obtain depth map and strength diagram of three-dimensional objects, and then the computational hologram of three-dimensional objects is obtained by combined with chromatography. The second was the iterative Fourier transform algorithm of the two-dimensional plane. But, to be able to achieve the effect of fast and clear calculation of the hologram of three-dimensional objects, after the three-dimensional objects were layered by the above-described chromatography. Based on the combination of chromatography and iterative Fourier transform algorithm these two algorithms, a quadratic depth phase factor was proposed. By encoding the secondary depth phase factor into the iterative operation of the iterative Fourier transform algorithm, it was possible to reflect the depth of the three-dimensional object. This algorithm is fast in operation, reproduction effect is good. Finally, through the experiment show that the method is viable.

Keywords: computational holography, three-dimensional representation, iterative Fourier transform, quadratic phase factor.

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I. INTRODUCTION

Nowadays, the rapid development of holographic technology is being used more and more widely in various occasions. From the initial optical holographic development to digital holography, and then the development of computational holography witnessed the progress of holographic technology. However, due to the relatively high requirements of experimental optical holography and digital holography, so the computational holography can be developed more and more rapidly. Computational holography also has a particularly large number of advantages, such as high precision, low noise, strong anti-interference and functional flexibility and so on. Moreover, computational holography can use computer software to directly handle the wave function, which can replace the traditional process of experimenting with optical devices to generate holograms directly and reproduce the results. In recent years, computational holography of three-dimensional display technology has made continuous breakthroughs, there have been products on the market, such as three-dimensional holographic television.

Because of the complex structure of three-dimensional objects and the huge amount of information, so the process of making it is very complicated and slow, and the calculation accuracy can't meet the requirements. In general, the basic method for holographic reproduction is chromatography^[1-2]; Multi-perspective projection method^[3-4]; Fresnel wave band method^[5]; Point source method^[6], and there is also an iterative Fourier transform algorithm method^[7-8]. This article will focus on chromatography and iterative Fourier transform algorithms method.

Usually, it is a very difficult problem to obtain the information of three-dimensional objects when calculating the simulation. However, with the rapid progress of holographic technology, now there is a more convenient way to obtain three-dimensional information. First, get a three-dimensional object model by scanning with a three-dimensional scanner^[9], the obtained three-dimensional model is saved as .3ds format, and import 3DsMax three-dimensional software to obtain the object's intensity map and depth map, the intensity map and depth map will contain all the information of three-dimensional objects, including amplitude information and phase information. Through the above forms and methods, the reconstruction of any three-dimensional object can be realized, breaking the limitation that the traditional computing holography only relies on the establishment of a model.

Among the above several basic methods, the holographic computation of three-dimensional objects is very slow and a good reproduction effect can't be achieved. In order to be able to quickly and clearly calculate the three-dimensional object hologram, we first use the above method to obtain the information of

three-dimensional objects, then based on the original iterative Fourier transform algorithm, after analyzing the properties of the lens^[10], we concluded the quadratic depth phase factor, in this way, the depth characteristics of the three-dimensional object are obtained, so that the three-dimensional object can be quickly and clearly reproduced.

II. CHROMATOGRAPHY

1. Based on two-dimensional plane's chromatography

The method of chromatography is to divide the three-dimensional objects along the depth direction Z, parallel and equidistant into multiple two-dimensional planes. Then, the each layer of two-dimensional plane information is diffracted by the Fresnel diffraction and then the complex amplitude is superposed. Finally, the hologram of the original three-dimensional object is obtained by adding the reference light. Figure.1 shows the chromatographic principle diagram.

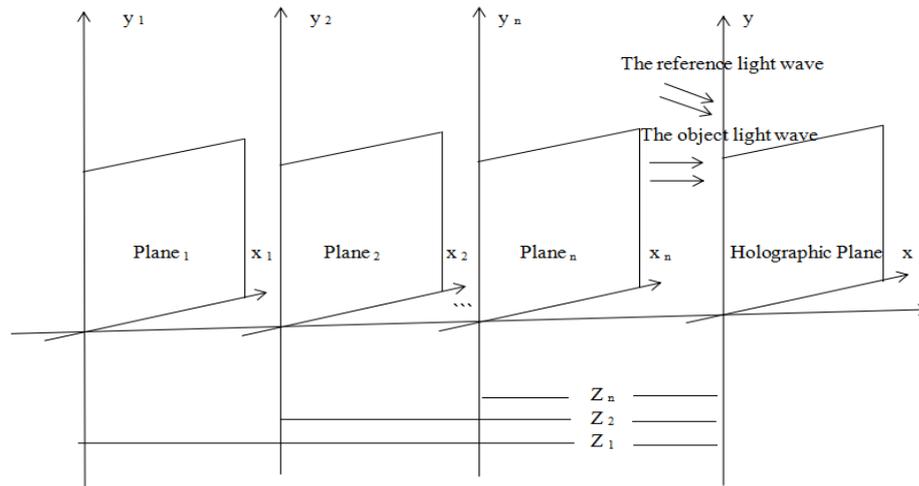


Figure.1 Chromatographic principle diagram

Assuming that a three-dimensional object is divided into n layers in parallel and equidistant. Currently taken as the i layer, assuming that the diffraction distance from the i-th layer plane to the holographic surface is z_i , then the complex amplitude of the i-th layer plane is expressed as:

$$U_i(x, y) = \text{FFT}^{-1}\{\text{FFT}[A_i(x, y)] \times \exp[jkz_i(x^2 + y^2)]\} \quad (1)$$

Which $A_i(x, y)$ represents the complex amplitude of the light field of i layer, $U_i(x, y)$ represents the complex amplitude of a hologram of i layer.

It follows that the total complex amplitude distribution is as follows:

$$U(x, y) = \sum_{i=0}^n U_i(X, Y) = A(x, y) \times \exp[jkz_i\phi(x, y)] \quad (2)$$

Adding reference light for diffraction. The reference light expression is:

$$R(x, y) = B(x, y) \times \exp[jkz_i\phi(x, y)] \quad (3)$$

In the experiment, the pixel of two two-dimensional plane images are 512×512. In Figure.2, the reproduction distance of light word picture to the holographic surface is 200mm, the reproduction distance of the letter picture to the holographic surface is 250 mm. After the complex amplitude of the two images are superimposed, adding a reference light to make a total hologram. We reproduced at 200mm and 250mm distances respectively, Figure.3 shows the reproduction image at 200mm and 250mm distance, at the distance of 200mm, the light word in Figure 3 is clearly, and at the distance of 250mm, the letter in Figure 3 is clearly. From this summary: Image reproduce at different distances and corresponding distance is clear, but image is blurry at another reproduction distance. It is proved that the image reproduction with chromatography under different depths can be realized, and it is also possible to deduce that the three-dimensional object can be reproduced.



Figure2. Two-dimensional plane picture

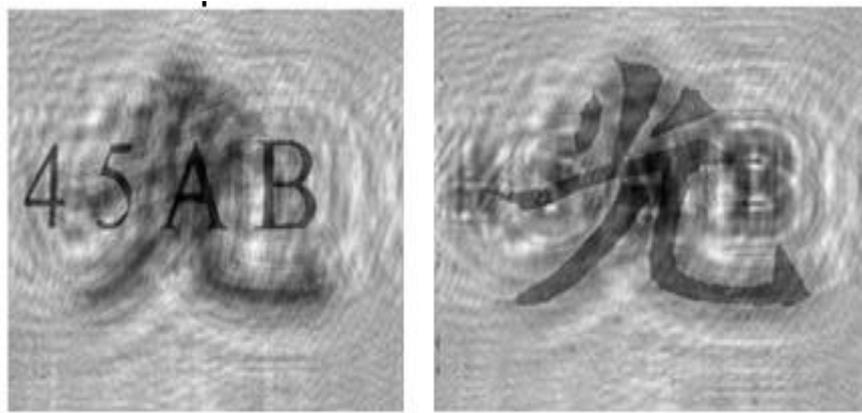


Figure3. Reproduction pictures at the distance of 200mm and 250mm

Chromatography Based On Depth Map And Intensity Map

Holographic technology not only needs to be used in the simulation of three-dimensional model, but also needs to be used in the actual object, daily real objects can be scanned by the 3D scanner to obtain the model, the resulting file is saved as .3ds format, importing the model into 3DsMax and using 3DsMax rendering technology, adding "z-depth" to the scum element in the option Render Elements. Then choosing to set the batch rendering, adding and saving the intensity map and the depth map's output path, outputting the intensity map and depth map. MATLAB software can be used to read the grayscale depth map information, in addition, the difference in the size of grayscale in depth maps can indicate the difference in depth of objects, the same object surface depth value is the same size. Assuming that the maximum value of depth value obtained is z_{max} and the minimum value is z_{min} , then the depth interval of each two images is taken as D , the total number of object surfaces of the three-dimensional object can be obtained by the formula $N = (z_{max} - z_{min}) / D$.

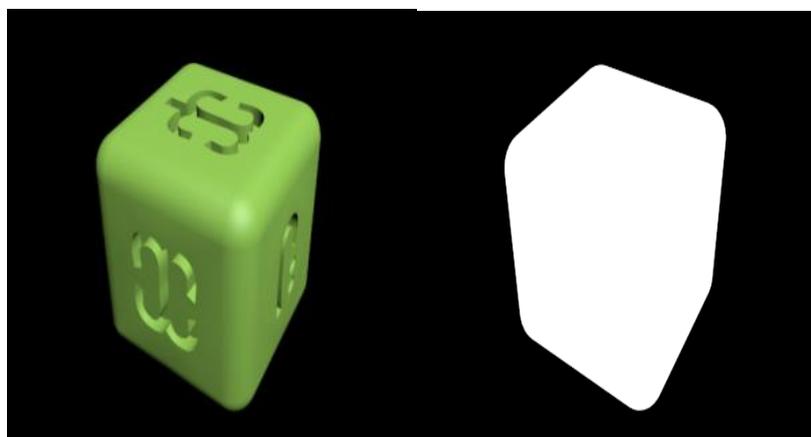


Figure4. Intensity picture and depth picture

The resulting cube's intensity and depth maps in Figure.4 have a resolution of 457×333 . We can know that the maximum value of depth map depth value is 457 through MATLAB software, the minimum value of 287, the depth interval D is 0.2mm, by the above formula, the cube should be divided into 85 layers of two-dimensional plane. We assume that cube satisfies the distance from Fresnel diffraction to holographic surface is $z_0 = 250 \text{ mm}$, then the Fresnel diffraction distance can be obtained by the size of the gray value in the depth map, the formula is $z_i = z_0 - i * D_i$. The magnitude of the intensity value can be calculated according to the coordinates of the location where the grayscale value of each plane is located, after the calculation of Fresnel diffraction, the complex amplitude of each layer plane is superimposed. Finally, a complete composite hologram is obtained by adding the reference light for diffraction.

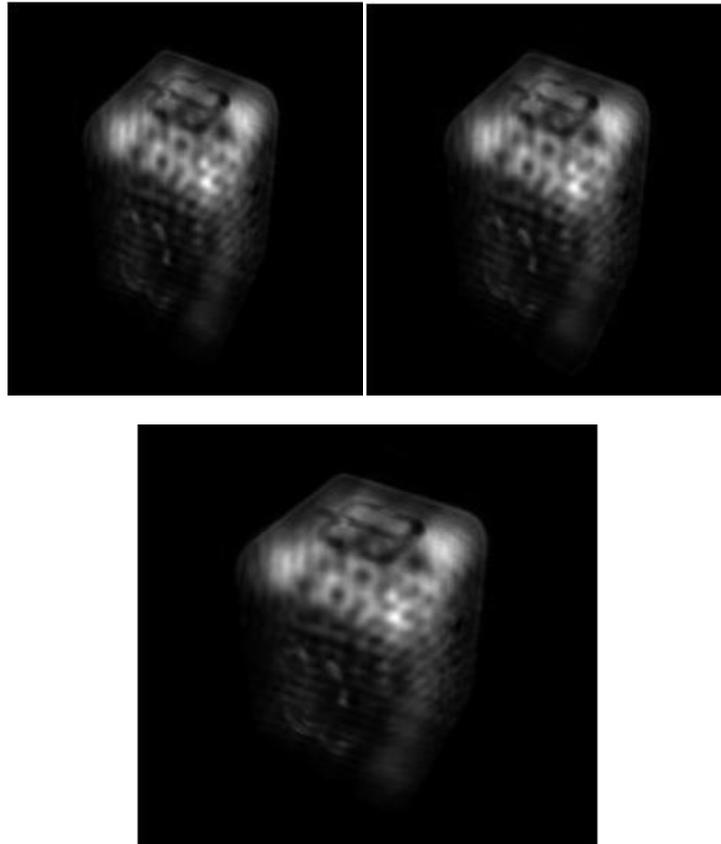


Figure5. Reproduction pictures at a distance of 250mm, 252.4mm, 248.6mm

It can be seen from figure.5 that the reproduced image of the object at the reproduction distances of 250 mm, 252.4 mm and 248.6 mm, respectively, we can get the best reproduction image quality at distances of 252.4 mm.

Iterative Fourier transform Algorithm

1. Two-dimensional hologram's iterative Fourier transform calculation

GERCHBERG and SAXTON, who proposed an iterative Fourier transform algorithm (GS) to calculate the two-dimensional plane's hologram. The essence of this algorithm is an iterative process. The central idea is to perform the Fourier transform and the inverse Fourier transform after replacing the intensity mode and the expected mode of the input wave in each iteration step while keeping the phase distribution constant. Therefore, the initial input wave and the unknown phase on the final target plane will eventually both be calculated. As shown in Figure.6, the element of the phase constraint in the GS algorithm flow chart is phase (the phase hologram) and the amplitude is normalized to 1 at each iteration. The constraint of amplitude matches the amplitude of the reconstructed object with the expected input amplitude, so as to keep the phase invariant. By judging whether the value of the root mean square error is smaller than the set value, if less than the set value, then the operation is finished, the binary hologram is outputted, and the reproduced object is checked.

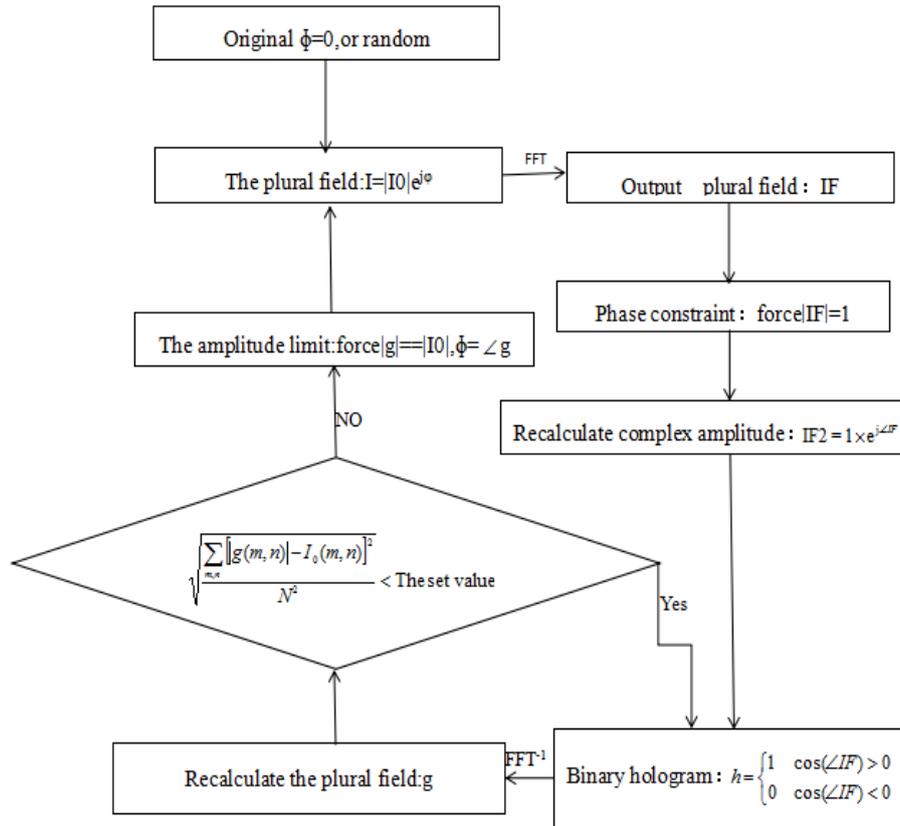


Figure6. GS Algorithm flow diagram

As shown in Figure.7, reconstruction of the original two-dimensional plane is performed after Fourier iteration. It can be seen from the figure that the reproduction effect is very good. The similarity between the reconstructed image and the target image is achieved after iterative Fourier operation.



Figure7. Original picture and reconstructed image

A New Method Of Three-Dimensional Hologram Calculation Based On Iterative Fourier Transform Algorithm

Holographic lenses have the following properties: The transmission factor in the lens contains the quadratic phase factor, which will result in the phase modulation of the incident wavefront. When the hologram is behind the lens, the optical path is shown in Figure.8.

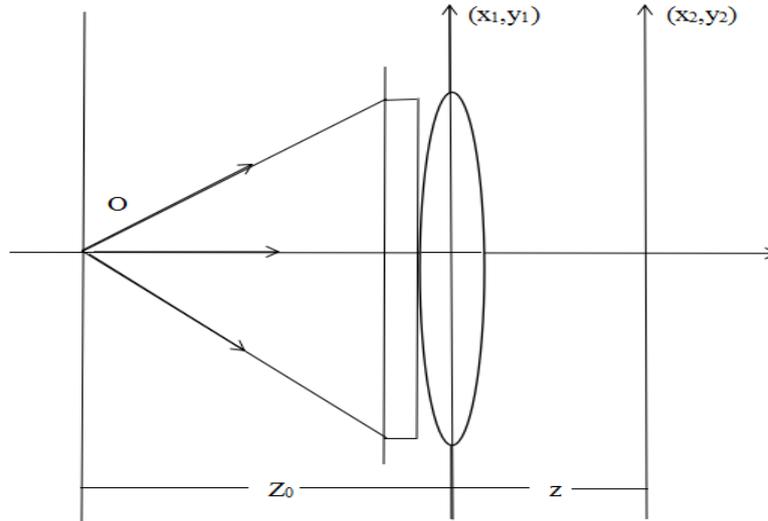


Figure8. Optical path diagram

z_0 represents the distance from the point light source to the lens, z represents the distance from the lens to the holographic imaging plane, we assume that the lens focal length is f , so the three have the following relationship:

$$\frac{1}{f} = \frac{1}{z_0} + \frac{1}{z} \tag{4}$$

The optical field on the right side of the holographic image is distributed as:

$$U_1(x, y) = O(x_1 + y_1) \times \exp\left[jk \frac{(x_1 + y_1)^2}{2z_0}\right] \tag{5}$$

After the light field to the right of the holographic imaging plane propagates by distance z , the distribution of the light field after reaching the back focal plane of the lens is:

$$\begin{aligned} U(x, y) &= C \exp\left[jk \frac{(x_2^2 + y_2^2)}{2z}\right] \times \iint_{-\infty}^{+\infty} O(x_1, y_1) \exp\left[jk \frac{(x_1^2 + y_1^2)}{2z_0}\right] \times \exp\left[-jk \frac{x_1 x_2 + y_1 y_2}{z}\right] dx_1 dy_1 \\ &= C \exp\left[jk \frac{(x_2^2 + y_2^2)}{2z}\right] \times \text{FFT}\{O(x_1, y_1) \times \exp\left[\frac{(x_1^2 + y_1^2)}{2z_0}\right]\} \end{aligned} \tag{6}$$

The coordinates (x_1, y_1) are holographic plane, the coordinates (x_2, y_2) is the image of the back focal plane of the lens. From the above relationship can know, due to the existence of the lens, the amplitude of the reproduced light field has two more quadratic phase factors than the original hologram Fourier transform, the phase distribution of the reproduction light field will be affected by the previous quadratic phase factor, the holographic coordinates are related to the next quadratic phase factor, the last quadratic phase factor is defined as the quadratic phase factor, the expression is as follows:

$$D = \exp\left[jk \frac{(x_1^2 + y_1^2)}{2z_0}\right] \tag{7}$$

In the experiment, we can use the concave lens to diverge the parallel light, if we add a variety of concave lens of different focal lengths, from the relationship of the three variables in Eq.(4), point light sources at different distances can replace by experimental parallel light, this is changing the distance z_0 , and thus the imaging distance z will change, in order to show the depth changes of three-dimensional objects. According to the above summary: Throughout the imaging process, if the parallel optical amplitude constraint is added, then the corresponding quadratic depth phase factor is multiplied, the inverse Fourier transform is divided by the corresponding quadratic depth phase factor, this shows the effect of different depth distances on the calculation of 3D holograms, resulting in depth information.

Based on this, it can be shown that the depth characteristics of the three-dimensional object can be obtained by encoding into the quadratic depth phase factor. The above-mentioned chromatographic method is combined with this method, and the object is cut into two-dimensional planes one by one in the depth direction, steps such as the following algorithm flow:

- 1) According to the principle of chromatography cutting an object along the depth direction, cutting method reference above, can be obtained diffraction distance z_i .
- 2) According to the diffraction distance isz_i , we can find that the corresponding point light source to the lens distance isz_0 , and calculate the corresponding quadratic depth phase factor D_i from z_0 .

- 3) Generating a random phase φ_0 , and using φ_0 as the initial phase of the hologram.
- 4) If the number of iterations at this time is 1, then skip this step, if this time is not 1, determining whether the number of iterations is the maximum, if yes, outputting the result at this moment, if not, continuing.
- 5) After adding the amplitude limit of parallel light, multiplying the corresponding quadratic depth phase factor D_i , and then performing Fourier transform to obtain the current reproduced image complex amplitude.
- 6) After adding the amplitude limit to the current reproduced image, we assume that the resulting phase is φ_i , then the amplitude of the image plane multiplied by the phase is the complex amplitude of the reconstructed image surface, the expression is: $H_i = A_i e^{\varphi_i}$.
- 7) Doing inverse Fourier transform on complex amplitudes which we got, then dividing the corresponding quadratic depth phase factor D_i .
- 8) The complex amplitude H_i of each image plane is superimposed, the result is complex amplitude H , taking the phase as φ , repeating the step 5.

The figure below shows a three-dimensional model of the head, importing it into 3DMax to get the depth map and intensity map. It is assumed that the model satisfies the distance from Fresnel diffraction to holographic surface is $z_0 = 250\text{mm}$, the pixel of the depth map and intensity map which we got are 1920×1080 , by MATLAB algorithm, we can know that the maximum value of depth value of this model is 230, the minimum value is 36. The interval is $D = 0.2\text{mm}$, so this model will be divided into $N = (230 - 36) / 2 = 97$ layers, and diffraction distance from the two-dimensional plane in each layer to the holographic plane is $z_i = z_0 - i * D_i$.

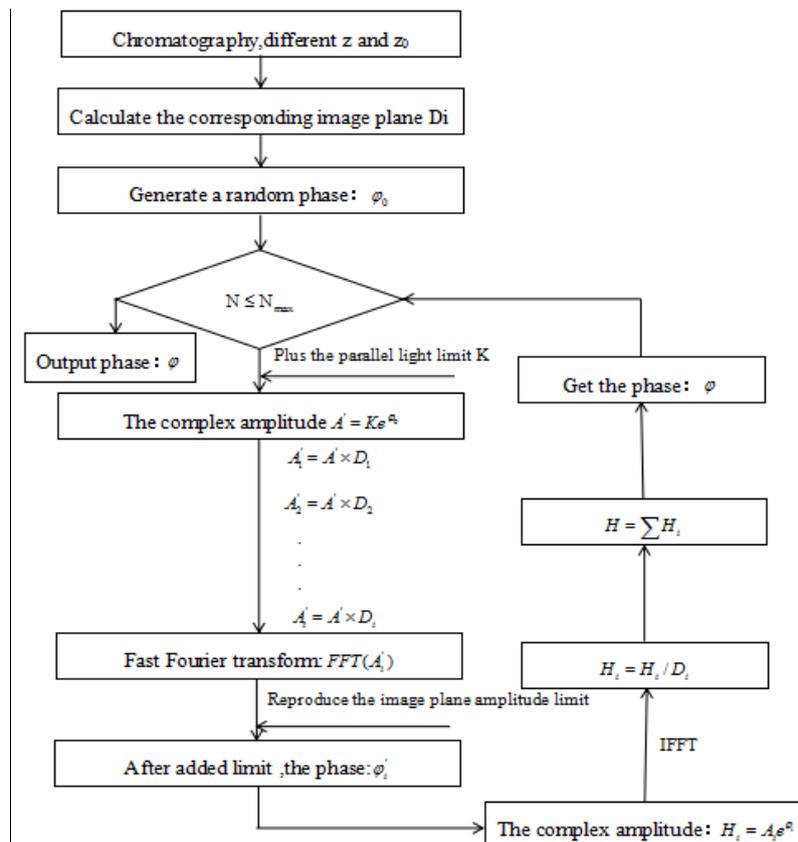
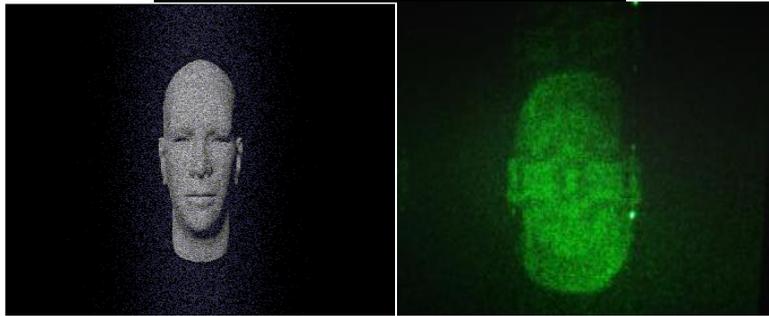
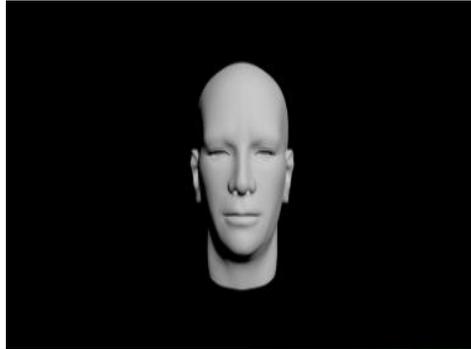


Figure9. Algorithm flowchart

III. EXPERIMENTAL RESULTS AND ANALYSIS

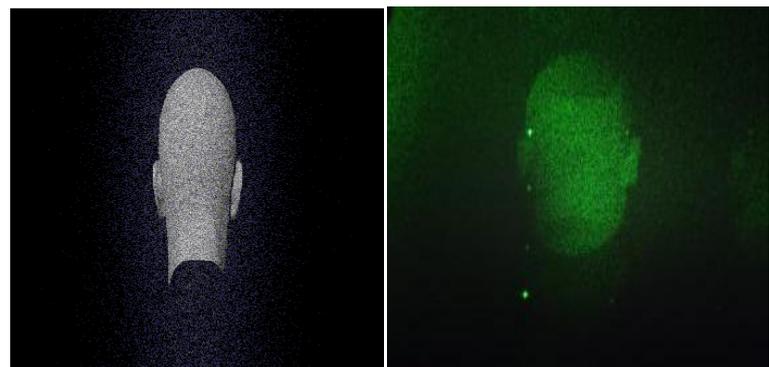
In this experiment, a pure phase spatial light modulator with resolution of 1920×1080 is adopted, the reproduction light used was green light of 52 nm. When using the above algorithm encoding, we should set the maximum number of iterations to 150 times, and operation time is 6.34s. The distance between the front surface of the model and the holographic surface is 250 mm, and because the depth of model is 194 mm, so the distance between the back surface and the hologram is 444 mm. We reproduce respectively in the reproduction distance of 250mm, 347mm and 444mm. As shown in Figure.10, through the simulation, we obtain the reproduced images and experimental reproduced images respectively at three different distances.



(a) Simulation reproduction distance is 250 mm; Experimental reproduction distance is 250 mm



(b) Simulation reproduction distance is 347 mm; Experimental reproduction distance is 347 mm



(c) Simulation reproduction distance is 444 mm; Experimental reproduction distance is 444 mm

Figure10. Three-dimensional model and Reproduction at different distances

By observing the above experimental phenomena can be knew:At 250mm reproduction distance,the front surface of the model is clearly imaged;At 347mm reproduction distance,it does not match the model's imaging surface,and occurs defocus,so imaging is fuzzy;At 444mm reproduction distance,the back of the model is clearly imaged.Accordingly, it is determined that the algorithm proposed in this paper can effectively record the depth information of objects.Clear imaging can be done under the corresponding surface of the model at various reproduction distances. We can conclude by analyzingfigure 11's root mean square error:As the number of iterations increases,the error decreases, meeting the requirement that the error value between the image and the target image can't exceed the maximum value after satisfying the iterative Fourier transform.

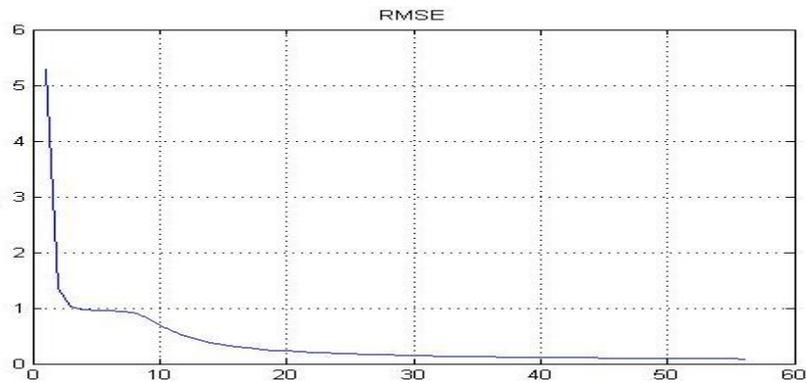


Figure 11. Root mean square error

IV. CONCLUSION

This article first introduced chromatography and chromatography based on depth and intensity maps of three-dimensional objects, then briefly introduced the iterative Fourier transform algorithm. Finally, by analyzing the Fourier transform properties of the lens, we concluded that there were quadratic depth phase factors, and combined with the above chromatography, the whole three-dimensional object was layered to obtain the diffraction distances at different levels, and quadratic depth phase factors corresponding to different diffraction levels are obtained. Then, quadratic depth phase factors were added into the iterative operation, those steps could achieve three-dimensional reproduction.

Experiments showed that this method could achieve relatively faster three-dimensional reproduction, and the reproduction effect was relatively good. Comparing to the traditional algorithm, the method proposed in this paper could achieve three-dimensional reproduction of any object, iterative arithmetic was performed on object plane of each layer of a three-dimensional object, this not only fast operation and high convergence. In the experiment, the time required to iterate 100 times and 150 times for a two-dimensional planar iterative Fourier transform is 7.440 and 11.205. The time required for this algorithm to iterate 100 and 150 times is 9.894 and 15.382. So we can draw the conclusion, the algorithm has also been greatly improved in the operation rate under the condition of improving the reproduction effect.

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