International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 4 Issue 8 ǁ August. 2016 ǁ PP. 08-14

Dimensional Constraint Type Mapping to Position and Orientation CharacteristicSet of PM

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Abstract: *POC setis used to describe the positionand orientationcharacteristic of the relative motion of two arbitrarilymoving links ofPM. The dimensional constraint type is one of the significant elements of the topological structure ofPM, and the topological structure characteristics of PM depend on its topological structure.This paper deals with the topological structure analytical of general 3-5RPM and two special 3-5$RPMs$ with the applications of the theory and methods for topological structure analytical of PMs based on POC set. The topological structure characteristics of these PMs is derived with this systematic theory and method; then, more detailed description are applied to the topological structure characteristics of two kinds of special 3-5$RPMs$. This research explains that the changes of the dimensional constraint type not only affect the POC Set of PM, but also lead to changes in DOF of PM.*

Keywords:- *Dimensional constraint type; POC set; 3-5$RPMs$*

I. INTRODUCTION

Over the last decade, the major topological structure synthesis theory for parallel mechanisms (PMs) has been established home and abroad. They are the ScrewTheory[1], the Displacement Subgroup Theory[2-4], and the Position and Orientation Characteristic (POC) Theory[6].

The analytical theory and methods of the topological structure of mechanism based on POC set,putting out by Professor Yang and others, features the following characteristics: non-instantaneity of mechanism andDOF, easier mathematical calculation and generality of mechanism. Moreover, they put out the significant concept of dimensional constraint type, which is used to describe the geometric constraint types of relative position and orientation between the axis of adjacent kinematic joints on one chain(including parallel, coaxial and insert at one point). Three key elements of topological structure of mechanism are dimensional constraint type、kinematic joint type and connection relationship of structural unit, which is one of the theoretical basis of constructing equation and operational rules of the POC set of mechanism.

Adopt analytical theory and methods of the topological structure ofPM based on POC set and take general and two special 3-5RPM for studying objective. By analyzing and study the impact of dimensional constraint type on POC set andDOF of PM,we will further reveal mapping relation between dimensional constraint type and POC set of mechanism. On this basis, we will give more detailed description on the features of the topological structure of two special 3-5RPM, thus we will expand the applications of POC set in analyzing the topological structure ofPM.

II. DIMENSIONAL CONSTRAINT TYPE AND DOF FORMULA

2.1Definition of the dimensional constraint type

The dimensional constraint type means the geometric constraint type in the relative orientation between the axis of the kinematic joints, which nature is that the topological structure of mechanism is introduced into the dimensional parameter type(the rod(axis) length is zero or any non-zero value, the twist angle is zero or $\pi/2$ or any non-zero value and so on). The following is the dimensional constraint type existing in the axis of the revolute

(R) joint of the branch in 3-5RPMs:

(1) Axis of the several adjacent R joint parallel to each other, marked as: $SOC\{-R \parallel R \parallel \cdots \parallel R-\}$;

(2) Axis of the two adjacent R joint coincide with each other, marked as: $SOC\{-R|R-\}$;

(3) Axis of the several adjacent R joint intersect at one point, marked as: $SOC\left\{-\overbrace{RR\cdots R}-\right\}$;

(4) General type, marked as: $SOC\{-R-R-\cdots R-\}$.

2.2 DOFformula

To reveal the inner relationship of the topological structure of PMs、the degree of freedom (DOF) and POC set, more general DOF formula (proposed by Yang and Sun) is applied as follow[12-15]:

$$\begin{cases} F = \sum_{i=1}^{m} f_i - \sum_{j=1}^{v} \xi_{L_j} \ (1-1a) \\ \xi_{L_j} = dim.\left\{\left(\bigcap_{i=1}^{j} M_{b_i} \cup M_{b_{(j+1)}}\right)\right\} (1-1b) \end{cases} \quad (1-1)$$

Where

f_i:DOF of the ith R joint;

m: Number of kinematic joint;

v: Number of independent loops ($v = n - m + 1$);

$\bigcap_{i=1}^{j} M_{b_i}$: POC set of the moving platform forsub-PM, formed by the front $j$branches;

M_{b_i}: POC set of the end link of the ith branch;

ξ_{L_j} : Number of independent displacement equations for the j th branch, which consists of equivalent single-open-chain (SOC) of the moving platform for sub-PM, formed by the front $j$branches, and the $(j+1)$th branch.

III. TOPOLOGICAL STRUCTURE AND DOF OF GENERAL 3-5RPM

General 3-5RPM shown in Fig.1, in which R denotes revolute joint. In thePM, triangle $A_1A_2A_3$ and $B_1B_2B_3$ are equilateral triangles; between the base and the moving platform, the three branches are connected in parallel; each branch is connected in series with 5 R joints, and all the R joints are arbitrary.

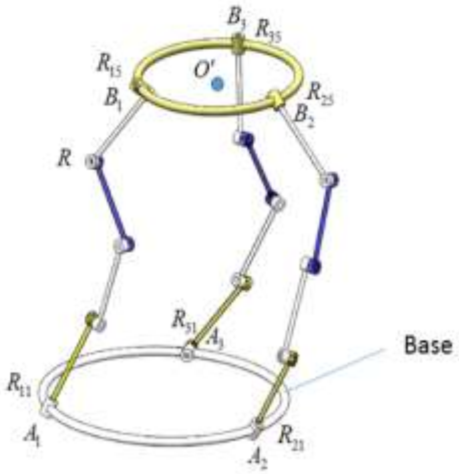

Fig.1. General 3-5RPM

3.1POC set of general 3-5RPM

In Fig.1, the dimensional constraint type of general 3-5RPM belongs to General type, marked as $SOC\{-R_{i1}-R_{i2}-R_{i3}-R_{i4}-R_{i5}\}(i=1,2,3)$. We can easily know that the branch's DOF is 5, therefore, the POC set of the end link is

$$M_b = \begin{bmatrix} t^2 \\ r^3 \end{bmatrix} \text{ or } M_b' = \begin{bmatrix} t^3 \\ r^2 \end{bmatrix}.$$

Then, the DOF of general 3-5RPM is calculated easily by using the formula $(1-1)$ and the way based on POC set.

Due to the dimensional constraint type of the general 3-5RPM branch belongs to General type and the formula$(1-1b)$, the number of independent displacement equations ξ_{L_1} for the first branch is

$$\xi_{L_1} = \dim.\left\{M_{b_1} \bigcup M_{b_1}\right\} = \dim.\left\{\begin{bmatrix} t^3 \\ r^3 \end{bmatrix}\right\} = 6$$

With the formula$(1-1a)$, the DOF of sub-PM consisting of the first and second branches is

$$F_{(1-2)} = \sum_{i=1}^{m} f_i - \sum_{j=1}^{1} \xi_{L_j} = 10 - 6 = 4$$

Similarly,

$$\xi_{L_2} = \dim.\left\{M_{pa\,(1-2)} \bigcup M_{b_3}\right\} = \left\{\begin{bmatrix} t^3 \\ r^3 \end{bmatrix}\right\} = 6$$

$$F = \sum_{i=1}^{m} f_i - \sum_{j=1}^{2} \xi_{L_j} = 15 - (6+6) = 3$$

Therefore, the DOF general 3-5RPM is 3, and the POC set of its moving platform is

$$\begin{bmatrix} t^3 \\ r^3 \end{bmatrix},$$

which has three independent elements. It is impossible to be sure about the nature of six elements, because the dimensional constraint type of general 3-5RPM belongs to General type. Therefore, it is impossible to be sure about the motion mode of the moving platform of general 3-5RPM, and it could not describe it more detailed.

I. Topological structure and DOF of two special PMs

By giving an introduction to two special 3-5RPM, we go further discussion on the impact of the changes of dimensional constraint type on the POC set of 3-5RPM. Special 3-5RPM is actually a new mechanism derived from the change of the dimensional constraint type (arbitrary position between R axis change to a state of parallel, collinear or insert at one point) of general 3-5RPM.

During this process, POC set of PM and the features of the topological structure change. To be more specific, the change may be the motion mode of the moving platform of PM or the change of DOF and motion mode.

4.1 POC set of 3-RERPM

3-RERPM[9] shown in Fig.2. On the base $A_1A_2A_3$, R_{11}、R_{21} and R_{31} axis parallel to each other, and perpendicular to the base plane; on the moving platform $B_1B_2B_3$, R_{15}、R_{25} and R_{35} axis parallel to each other, and perpendicular to the moving platform plane; three R joints in the middle of the branch parallel to each other, and form a E joint; R_{11} and R_{15} axis parallel to each other.

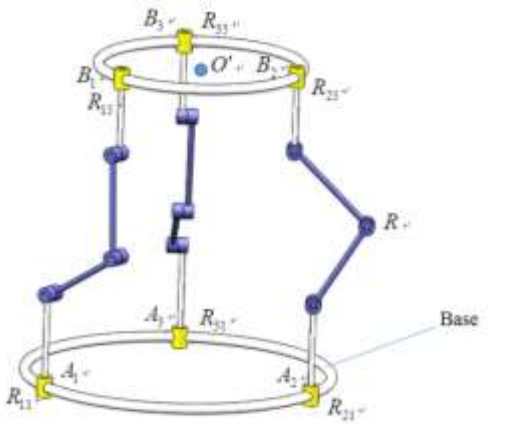

Fig.2.3-RERPM

Topological structure of the PM is analyzed by using the theory and methods based on POC set. Thus POC set and topological structure of the PM are obtained.

(1) Topological structure of PM

a. Topological structure of branch:

Three branch:SOC$\{-R_{i1}-R_{i2}//R_{i3}//R_{i4}-R_{i5}-\}(i=1,2,3)$;

$R_{i1}//R_{i5}(i=1,2,3)$.

b. Topological structure of two platform:

The base: R_{11}、R_{21} and R_{31} axis parallel to each other, and perpendicular to the base plane;

The moving platform: R_{15}、R_{25} and R_{35} axis parallel to each other, and perpendicular to the moving platform plane.

(2) Select the midpoint of the moving platform $B_1B_2B_3$ as the basic point O'

(3) Determine the POC set of the branch end link

Combined with the dimensional constraint type of the branch, the POC set of the branch end link is

$$M_{b_i}=\begin{bmatrix}t^1(\perp R_{i1})\\r^1(//R_{i1})\end{bmatrix}\bigcup\begin{bmatrix}t^2(\perp R_{i2})\\r^1(//R_{i2})\end{bmatrix}\bigcup\begin{bmatrix}t^1(\perp R_{i5})\\r^1(//R_{i5})\end{bmatrix}=\begin{bmatrix}t^3\\r^2(//\diamond(R_{i1},R_{i2}))\end{bmatrix}(i=1,2,3)$$

(4) Determine the number of independent displacement equationsξ_{L_1} for the first branch

a. with the formula $(1-1b)$, the number of independent displacement equations ξ_{L_1} for the first loop is

$$\xi_{L_1}=dim.\left\{M_{b_1}\bigcup M_{b_2}\right\}=dim.\left\{\begin{bmatrix}t^3\\r^2(//\diamond(R_{11},R_{12}))\end{bmatrix}\bigcup\begin{bmatrix}t^3\\r^2(//\diamond(R_{21},R_{22}))\end{bmatrix}\right\}=dim.\left\{\begin{matrix}t^3\\r^3\end{matrix}\right\}=6$$

b. with the formula $(1-1a)$, the DOF of sub-PM consisting of the first and second branches is

$$F_{(1-2)}=\sum_{i=1}^{m}f_i-\sum_{j=1}^{1}\xi_{L_j}=10-6=4$$

c. considering that R_{11} and R_{21} axis parallel to each other, and R_{12} axis is not parallel to R_{22} axis, POC set of the sub-PM consisting of the first and second branchs is

$$M_{pa(1-2)}=M_{b_1}\bigcap M_{b_2}=\begin{bmatrix}t^3\\r^2(//\diamond(R_{11},R_{12}))\end{bmatrix}\bigcap\begin{bmatrix}t^3\\r^2(//\diamond(R_{21},R_{22}))\end{bmatrix}=\begin{bmatrix}t^3\\r^1(//R_{11})\end{bmatrix}$$

(5) Determine the number of independent displacement equations ξ_{L_2} for the second branch

With the formula $(1-1b)$, the number of independent displacement equations $\xi_{L_2}$for the second loop is

$$\xi_{L_2}=dim.\left\{M_{pa(1-2)}\bigcup M_{b_3}\right\}=dim.\left\{\begin{bmatrix}t^3\\r^1(//R_{11})\end{bmatrix}\bigcup\begin{bmatrix}t^3\\r^2(//\diamond(R_{31},R_{32}))\end{bmatrix}\right\}=dim.\left\{\begin{bmatrix}t^3\\r^2\end{bmatrix}\right\}=5$$

(6) Determine the DOF of the PM

With the formula $(1-1a)$,

$$\mathrm{F} = \sum_{i=1}^{m} f_i - \sum_{j=1}^{2} \xi_{L_J} = 15 - (6+5) = 4$$

(7) Determine POC set of the PM

$$\mathrm{M}_{pa} = \mathrm{M}_{pa(1-2)} \bigcap \mathrm{M}_{b_3} = \begin{bmatrix} t^3 \\ r^1(//\mathrm{R}_{11}) \end{bmatrix} \bigcap \begin{bmatrix} t^3 \\ r^2(//\diamondsuit(\mathrm{R}_{31},\mathrm{R}_{32})) \end{bmatrix} = \begin{bmatrix} t^3 \\ r^1(//\mathrm{R}_{11}) \end{bmatrix}$$

(8) Motion characteristic analysis of the moving platform

With M_{pa} and $\mathrm{DOF} = 4$, we can know that the 3-RERPMhas three independent translation and an independent rotation (parallel to R_{11} joint).

4.2 POC set of 2ERR-RERPM

2ERR-RERPM[10] shown in Fig.3. On branches $\mathrm{A}_1\mathrm{B}_1$、$\mathrm{A}_3\mathrm{B}_3$, the front three R joints parallel to each other and the back two R joints parallel to each other, and the two sets of R axis is not parallel to each other; inbranch $\mathrm{A}_2\mathrm{B}_2$, the middle three R joints parallel to each other.

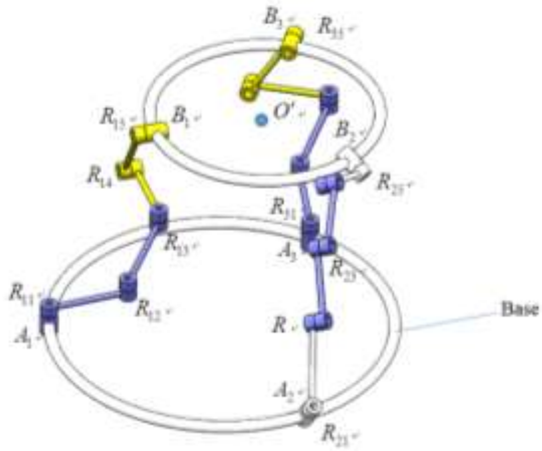

Fig.3.2ERR-RERPM

Topological structure of the PM is analyzed by using the theory and methods based on POC set. Thus POC set and topological structure of the PM are obtained.

(1) Topological structure of PM

a. Topological structure of branch:

Three branch: $\mathrm{SOC}\{-\mathrm{R}_{i1}//\mathrm{R}_{i2}//\mathrm{R}_{i3} - \mathrm{R}_{i4}//\mathrm{R}_{i5} -\}(i = 1,3)$;

$\mathrm{SOC}\{-\mathrm{R}_{21} - \mathrm{R}_{22}//\mathrm{R}_{23}//\mathrm{R}_{24} - \mathrm{R}_{25} -\}$.

b. Topological structure of two platform:

The base: R_{11}、R_{21} and R_{31} axis are space arbitrary intersection;

The moving platform: R_{15}、R_{25} and R_{35} axis are also space arbitrary intersection.

(2) Select the midpoint of the moving platform as the basic point O'

(3) Determine the POC set of the branch end link

Combined with the dimensional constraint type of the branch, the POC set of the branch end link is

$$\mathrm{M}_{b_i} = \begin{bmatrix} t^2(\perp \mathrm{R}_{i1}) \\ r^1(//\mathrm{R}_{i3}) \end{bmatrix} \bigcup \begin{bmatrix} t^1(\perp \mathrm{R}_{i4}) \\ r^1(//\mathrm{R}_{i4}) \end{bmatrix} = \begin{bmatrix} t^3 \\ r^2(//\diamondsuit(\mathrm{R}_{i3},\mathrm{R}_{i4})) \end{bmatrix} (i = 1,3)$$

$$\mathrm{M}_{b_2} = \begin{bmatrix} t^1(\perp \mathrm{R}_{21}) \\ r^1(//\mathrm{R}_{21}) \end{bmatrix} \bigcup \begin{bmatrix} t^2(\perp \mathrm{R}_{22}) \\ r^1(//\mathrm{R}_{23}) \end{bmatrix} \bigcup \begin{bmatrix} t^1(\perp \mathrm{R}_{25}) \\ r^1(//\mathrm{R}_{25}) \end{bmatrix} = \begin{bmatrix} t^2(\perp \mathrm{R}_{22}) \\ r^3 \end{bmatrix}$$

We can know that DOF of the PM is five, and POC set has five independent elements.

(4) Determine the number of independent displacement equations ξ_{L_1} for the first branch

a. with the formula $(1-1\mathrm{b})$, the number of independent displacement equations ξ_{L_1} for the first loop is

$$\xi_{L_1} = dim.\left\{\mathrm{M}_{b_1} \bigcup \mathrm{M}_{b_3}\right\} = dim.\left\{\begin{bmatrix} t^3 \\ r^2(//\diamond(\mathrm{R}_{13},\mathrm{R}_{14})) \end{bmatrix} \bigcup \begin{bmatrix} t^3 \\ r^2(//\diamond(\mathrm{R}_{33},\mathrm{R}_{34})) \end{bmatrix}\right\} = dim.\left\{\begin{bmatrix} t^3 \\ r^3 \end{bmatrix}\right\} = 6$$

b. with the formula $(1-1a)$, the DOF of sub-PMconsisting of the first and third branches is

$$\mathrm{F}_{(1-3)} = \sum_{i=1}^{m} f_i - \sum_{j=1}^{1} \xi_{L_J} = 10 - 6 = 4$$

c. considering that$\diamond(\mathrm{R}_{13},\mathrm{R}_{14})$ is not parallel to $\diamond(\mathrm{R}_{33},\mathrm{R}_{34})$, POC set of the sub-PM consisting of the first and third branches is

$$\mathrm{M}_{pa\,(1-3)} = \mathrm{M}_{b_1} \bigcap \mathrm{M}_{b_3} = \begin{bmatrix} t^3 \\ r^2(//\diamond(\mathrm{R}_{13},\mathrm{R}_{14})) \end{bmatrix} \bigcap \begin{bmatrix} t^3 \\ r^2(//\diamond(\mathrm{R}_{33},\mathrm{R}_{34})) \end{bmatrix}$$

$$= \begin{bmatrix} t^3 \\ r^1\left(//\left(\diamond(\mathrm{R}_{13},\mathrm{R}_{14}) \bigcap \diamond(\mathrm{R}_{33},\mathrm{R}_{34})\right)\right) \end{bmatrix}$$

(5) Determine the number of independent displacement equations ξ_{L_2} for the second branch

With the formula $(1-1\mathrm{b})$, the number of independent displacement equations ξ_{L_2} for the second loop is

$$\xi_{\mathrm{L}_2} = dim.\left\{\mathrm{M}_{pa\,(1-3)} \bigcup \mathrm{M}_{b_2}\right\} = dim.\left\{\begin{bmatrix} t^3 \\ r^1 \end{bmatrix} \bigcup \begin{bmatrix} t^2 \\ r^3 \end{bmatrix}\right\} = dim.\left\{\begin{matrix} t^3 \\ r^3 \end{matrix}\right\} = 6$$

(6) Determine the DOF of the PM

With the formula$(1-1\mathrm{a})$,

$$\mathrm{F} = \sum_{i=1}^{m} f_i - \sum_{j=1}^{2} \xi_{L_J} = 15 - (6+6) = 3$$

(7) Determine POC set of the PM

$$\mathrm{M}_{pa} = \mathrm{M}_{pa\,(1-3)} \bigcap \mathrm{M}_{b_2} = \begin{bmatrix} t^3 \\ r^1\left(//\left(\diamond(\mathrm{R}_{13},\mathrm{R}_{14}) \bigcap \diamond(\mathrm{R}_{33},\mathrm{R}_{34})\right)\right) \end{bmatrix} \bigcap \begin{bmatrix} t^2(\perp \mathrm{R}_{22}) \\ r^3 \end{bmatrix}$$

$$= \begin{bmatrix} t^2(\perp \mathrm{R}_{22}) \\ r^1\left(//\left(\diamond(\mathrm{R}_{13},\mathrm{R}_{14}) \bigcap \diamond(\mathrm{R}_{33},\mathrm{R}_{34})\right)\right) \end{bmatrix}$$

(8) Motion characteristic analysis of the moving platform

With M_{pa} and $\mathrm{DOF}=3$, we can know that the 2ERR-RER PM has two independent translation (in the plane being perpendicular to R_{22} joint) and an independent rotation (parallel to$\mathrm{L} = (\diamond(\mathrm{R}_{13},\mathrm{R}_{14}) \cap \diamond(\mathrm{R}_{33},\mathrm{R}_{34})))$. If R_{22} axis parallel to$\mathrm{L} = (\diamond(\mathrm{R}_{13},\mathrm{R}_{14}) \cap \diamond(\mathrm{R}_{33},\mathrm{R}_{34}))$, topological structure characteristics of the moving platform of the PM can be made out more clearly.

II. CONCLUSIONS

(1) The changes of the dimensional constraint type of PM always lead to changes of POC set of PM and the features of topological structure. By applying POC settheory and methods, we can obtain the mapping relation between the changes of dimensional constraint type and the changes of the features of its topological structure.

(2) By taking general and two special 3-5RPM for example, this article elaborates that the theory and methods based on POC set can be applied to the analysis of the topological structure of PM. That is to say, the POC settheory and methods can provide more precise description on the features of the topological structure of PM.

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