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Review on Genetic Algorithm for Solving Nonlinear Equations

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Abstract: Many problems in engineering application finally can be summed up in solving nonlinear equations.The Traditional solving methods of Newton method and then derived the quasi-newton method, although have the characteristics of quick convergence and high precision,but the result is the local solution, not the global optimal solution.Therefore to seek a algorithm of obtaining the global optimal solution has always been the goal of the researchers.With the emergence of intelligent algorithm with global search ability, especially the development of the genetic algorithm (GA), more and more people consider combining intelligent algorithm and traditional algorithm of their respective advantages to invent a new hybrid algorithm to solve the nonlinear equations.A series of numerical experiments show that the new algorithm does in their respective applications to solve problems which the traditional algorithm can not solve.This paper introduced the general overview of these algorithms, finally, the genetic algorithm in the aspect of numerical optimization is discussed.

Keywords: Genetic algorithm;Nonlinear equations;Newton method;Quasi-newton method

I. INTRODUCTION

Nonlinear equations has been a tireless research subject for scholars, it is not just a mathematical need, but also the need of the engineering.To the problem of modern large-scale engineering,many applications boil down to the solution of the nonlinear equations the last, such as oil and gas geological exploration, power system optimization, nonlinear fracture problem, etc.The classic solution of nonlinear equations is proposed by Newton, 1669, then this method is called Newton's method.With the developing of study, the method also appears a series of improvements.More representative means is simply Newton method, quasi-newton method, trust region Newton method, damped Newton method, finite difference Newton's method and so on, but these improvements method do not consider the Jacobi matrix of the structure.Schubert, who is the first one that consider the nonlinear equations of Jacobi matrix structure in 1970,he put forward a sparse Newton method for solving asymmetric nonlinear equations.Later, Powell, Toint and Shanno has derived the optimum sparse quasi-newton correction formula respectively. In 1984, Steihuag put forward a new strategy to deal with sparse quasi-newton method, and the convergence of sparse quasi-newton method is proved by Marwil in 1979.

Newton's method has a drawback of large amount of calculation, in order to reduce the amount of calculation, Dembo put forward a kind of inexact Newton method for solving nonlinear equations, Dembo[1]and others have proved the local convergence of inexact Newton method, and this method has both the characteristics of rapid convergence of traditional Newton method, and its every step of the iterative can get a better control precision.For solving nonlinear equations, iterative control sequence is part of the researchers focused on, Eisenstat and Walker published a paper on the issue[2].The essence of the inexact Newton method is a kind of inner and outer iteration, in 2003, Zhengda Huang gave the second order convergence result of inexact Newton method and the corresponding error estimates in Banach space[3].Fenghong Yang also proposed a weighted average of inexact Newton method and applied to power flow calculation of large power system[4].In 2008, professor Hao Liu and professor Qin Ni introduced inaccurate jacobin matrix into inexact Newton method,and proposed a new method of solving nonlinear equations[5].2015,Guoqing Tan and Xiaofei Zhu proposed a four step eight order iterative method for solving nonlinear equations and proved this method has

eight order convergence[6].Liu Yang and Yanping Chen proposed a new global convergence Levenberg - Marquardt algorithm, they are not suppose Jacobi matrix under the condition of the local error bounds of nonsingular and prove its global convergence and local convergence characteristics, finally the numerical experiments also show that the effectiveness of the algorithm.

Through the tireless efforts of scientific researchers, although the traditional algorithm in the convergence speed and convergence precision has made great progress, but, these algorithms depend on initial value , it is easy to cause a local solution, not the global optimal solution. With the development of intelligent algorithm, more and more people are combining intelligent algorithm and traditional algorithm to design a new algorithm that is used to obtain the global optimal solution. Moyuan Bin, Dezhao Chen and Shangxu Hu used chaotic particle swarm optimization algorithm to solve nonlinear equations,and combined chaotic search mechanism with particle swarm optimization to obtain the optimal solution in a appropriate search direction , and finally, they established the model of the fatigue life of composite material structure with stress, temperature, humidity, proved the particle swarm algorithm for solving nonlinear equations availably[7].Dongdong Wang and Yongquan Zhou used artificial fish algorithm to solve the nonlinear equations, after validation of numerical experiments show this method has the characteristics of high precision and fast convergent,it provides a new reference for nonlinear equations to calculate the direction[8].

II. GENETIC ALGORITHM FOR SOVLING NONLINEAR EQUATIONS

In recent years, with the development of artificial intelligence algorithms, a growing number of scholars have put forward the method based on artificial intelligence algorithm for solving nonlinear equations, such as genetic algorithm, particle swarm optimization, artificial fish algorithm, ant colony algorithm, etc. These intelligent algorithms generally transformed nonlinear equations into function optimization problem, which features good stability, insensitive to initial value, without derivation, calculation speed, but also exist the disadvantage of inaccuracy. Genetic algorithm is a global search algorithm, it has strong search ability, but there are also shortcomings, such as slow convergence and local search ability is insufficient, the length of the encoded directly affects the accuracy of the results, etc. Therefore, many scholars are struggling to seek an improved algorithm.

2.1 Hybrid Algorithms Of Genetic Algorithm And Classical Iteration Method

Genetic algorithm can search the result of nonlinear equations in the whole domain space, while classical iteration method can search the result accurately near the point of convergence. If we can combine both , that can play the advantages of these two algorithm and achieve better effect. In the current literature,scholars put forward the following mixed strategy mainly.

Mingwang Zhao introduced Newton iteration as a genetic operators of the algorithm and iterative the progeny individuals with the probability P, the new iteration value instead of the parent to join their offspring. Then he applied the hybrid algorithm to solving compatible nonlinear equations problem and had been achieved good results[9].Yazhong Luo used real number coding, design a mosaic type hybrid operator genetic algorithm, their design idea is:choosing individuals completed crossover and mutation operation with a certain probability to optimize with classic algorithms, and the result of optimization as the new individual chromosomes, classic algorithms as a strong local search operator of genetic algorithm in the evolutionary process[10].Literature [11] search the local accuracy results with quasi-newton method in every generation optimal individual, and that the adaptive hybrid operator probability should be increased with the increase of evolutionary,finally, approach to a constant , the paper therefore puts forward the following formula：

Where T is the biggest generation of genetic algorithm is the current evolution generation, constant .Yingshuang Qu combine BFGS quasi Newton method with genetic algorithm, a mixed strategy is the same with Mingwang Zhao's method, but the article uses finite Markov theory to establish global convergence theorem of the hybrid algorithm of genetic algorithm and BFGS algorithm , which proves that retain the current best solution of the hybrid algorithm can converge to the global optimal solution[12].Peng Wang improved Yingshuang Qu's work further, mainly: (1) calculate each individual adaptive value, choose good individual performance in accordance with a certain probability to iterate with quasi-newton iteration; (2) iterate the individual only once, save the amount of calculation.Tao Li improved classic BFGS variable metric method and genetic algorithm to design a new algorithm, the algorithm is to use the improved genetic algorithm to calculate a set of solution, and then as the initial solution of modified BFGS method and genetic algorithm, the numerical examples verify the effectiveness of this new method[13].

2.2 The Hybrid Of Genetic Algorithm And Intelligent Algorithm

Zhenyue Fu and Shunfang Wang aimed at the expansion of the solution space , improved the traditional genetic annealing algorithm, introduced concurrent mechanism and the maximum heap, through the simulation showing this mechanism improved the performance of genetic annealing algorithm, accelerating the solving speed[14].Zhihong Zhang combined interval algorithm and genetic algorithm to design a kind of improved genetic algorithm to avoid the local escape and precocious phenomenon of genetic algorithm[15].Narayanan presented quantum genetic algorithm[16] ,Juan Du designed a kind of nonlinear equations solving method based on quantum genetic algorithm[17]. Hong Xu put forward an improved quantum genetic algorithm, this algorithm has good adaptability. In recent years, the immune genetic algorithm based on biological immune principles is becoming a hot research topic in the field of optimal design, the immune genetic algorithm described problems as antigen, the solution of problem is described as antibody, using the degree of affinity of antigen and antibody to express the degree of approximation of feasible solution and the optimal solution.Jingfang Wang put forward a immune genetic algorithm to solve nonlinear equations of material balance in hydrocarbon steam reforming reactor , this algorithm has introduced the gauss mutation and updating strategy based on antibody concentration regulation mechanism, guarantee the diversity of antibodies, to avoid the problem of genetic algorithm premature[18].

2.3 Other Improvement Strategies Of Genetic Algorithm

Lewei Yan and Shuhui Chen used the population isolation mechanism, the optimal retention policy, arithmetic crossover method and adaptive random mutation to improve genetic algorithm, and the improvement to enhance the local search ability of genetic algorithm[19].Yufeng Li used real number coding, he converted the solution of the equations to the maximum problems, designing a algorithm with the best individuals keep accelerating convergence strategy, the final results show this method is effective[20].Moreover,Lei Chen and Yongliang Huo improved a floating-point genetic algorithm, this method introduced the local search information into the genetic algorithm , by changing the mutation operator to adjust the search area, and then use local search information to improve the accuracy of solution, the numerical simulation results proved the rapid speed and accuracy of that kind of improvement method.Abolfazl Pourrajabian presented an algorithm based on genetic algorithm to apply to solving three typical groups of equations , finally through different validation functions to verify the effectiveness of the proposed algorithm and analyzed the influence of initial population size on the algorithm[21].Ricardo M.A.Silva designed a BRKGA algorithm can obtain multiple solutions to solve the nonlinear equations[22].Hongmin Ren introduced symmetric coordinate individuals to design a kind of algorithm that is used to approximate solution of nonlinear equations[23]. Junhong He used the

entropy function to convert nonlinear equations to minimization optimization problems, they combined respective advantages of quasi-newton method and genetic algorithm to design a kind of algorithm can improve the efficiency of the overall optimization[24].Xianbin Wang who designed a kind of hybrid genetic algorithm to calculate the vehicle dynamic balance , finally using duffing equation to verify the validity of the method[25].

III. FURTHER PERSPECTIVES

Nonlinear equations is the most intractable problems in computing, extracting the mathematical model of most practical engineering problems can be transformed into nonlinear equations. Traditional iteration method because of its sensitivity to initial value is seldom applied directly, as in recent years the development of artificial intelligence methods, using intelligent algorithm to solve the nonlinear equations becomes a major subject, such as genetic algorithm, particle swarm optimization algorithm, ant colony algorithm, differential evolution algorithm, etc. But the genetic algorithm is the most widely used at present. More and more people combined genetic algorithm and the traditional classical algorithm to design a series of new algorithms, the algorithm also has the global search capability of genetic algorithm and classical algorithm of local optimization performance. After many years of research, using the genetic algorithm for nonlinear equations has had great development, but some problems are worth further studies. Topics of interest include the following-list of items

（1）The convergence theory of Genetic algorithm is worth studying. Goldberg and Segrest used markov chain to analyze the genetic algorithm, Rudolph used homogeneous finite markov chain to prove that the standard genetic algorithm can’t converge to the global optimal solution, the convergence of genetic algorithm has always been the research, in the future study, analysis of convergence of genetic algorithm is still a key point.

（2）The specific effect of mutation operator and exchange operator still needs further research. The selection of control parameters is also the focus of research in the future, because of the control parameters directly influence the nonlinear equations convergence .

（3）Genetic algorithm is applied to the specific engineering practice is also the trend of the future. At present, the domestic only stay in this level of thesis,it away from the actual product commercialization. Therefore, the genetic algorithm combines with specific engineering applications will be the future research direction.

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