International Journal of Research in Engineering and Science (IJRES)
ISSN (Online): 2320-9364, ISSN (Print): 2320-9356
www.ijres.org Volume 4 Issue 11 ǁ November. 2016 ǁ PP.67-71

Computation of the Scattering Cross-Section for Some Isospin States

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Abstract:- Different isotopes of medium and heavy beams from neutron poor to neutron rich in an energy range 30-100MeV cross section at 0^o is very important, its position in energy can be determined much more precisely experimentally than the absolute magnitude of the cross section. the energy minimum given by T_π = 45.1 6 ± 0.5 MeV was used in this work, using Fortran programmed KAP19 in this research to calculates the differential and the total scattering cross-section for the scattering of a neutron with energy E ≤ 50 MeV by a target nucleus with mass number A ≥ 40.. It was observed that in all the system considered there was resonance particle.

I. INTRODUCTION

Heisenberg suggested in 1932that the proton and neutron could be thought of as different statesof the same particle: `spin up' and `spin down' nucleon. This was the beginning of isospin (originally isotopic spin and sometimes isobaric spin) [1].Isospin is a continuous symmetry which the strong interaction does not distinguish between the neutron and proton. For example, the mass difference between the two is very small:

$$(m_n - m_p)/m_n \approx 10^{-3} \qquad 1$$

Heisenberg's thought was that if you could turn off electromagnetism then $m_p = m_n$. We now believe that that isospin symmetry is due the near equality of the up and down quarks ($m_u \approx m_d$)[3]. It was postulate that Isospin is conserved in the strong interaction, but not in the electromagnetic (or weak interaction). The strong interaction does not feel (or "couple") to electric charge so we expect the strong interaction of the proton and neutron to be the same. Thus the isospin operator (I) commutes with the strong Hamiltonian, but not the electromagnetic Hamiltonian[4].

$$[H_s, I] = 0 \text{ but } [\mathrm{H_{EM}}, I] \neq 0 \qquad 2$$

When constructing the wavefunction of a system under the strong interaction it is necessary to take isospin into consideration to make sure ithas the correct (boson or fermion) symmetry. This generalizes the Pauli Principle. [5-11].Isospin states are labeled by the total Isospin (I) and the third component of Isospin (I_3).When constructing baryons (3quark states) and meson (quark anti-quark states) account must be taken into the isospin of the quarks:

$$u-quark: I = 1/2, d-quark: I_3 = -1/2, \text{ all other quarks have I = 0} \qquad 3$$

Mathematically, Isospin is identical to spin, we combine Isospin the same way we combine angular momentum in quantum mechanics (QM). Like angular momentum, Isospin can be integral or half integral:

Particles	Total Isospin value (I)		
Λ^0 *or* Ω^-	0		
(p,n) or (K^0, K^+)	$1/2$	$(2I+1)$ *state*	4
$(\pi^+, \pi^0\pi^-)$	1		
$(\Delta^{++}, \Delta^+, \Delta^0, \Delta^-)$	$1/3$		

Like the proton and neutron, the three pion states (p^+, p^0, p^-) are really one particle under the strong interaction, but are split by the electromagnetic interaction. Just like ordinary angular momentum states. In this way of labeling we have:

Particles	Isospin state $\|I,I_3>$	
L^0 or W^-	$\|0,0>$	
proton or K^+	$\|1/2,1/2>$	
neutron or K^0	$\|1/2,-1/2>$	5

p^+	|1,1>
p^0	|1,0>
p^-	|1,-1>

The development of particle accelerators and the measurement of scattering cross sections revealed new particles in the form of resonances. The first resonance in particle was discovered by H. Anderson, E. Fermi, E. A. Long, and D. E. Nagle, working at the Chicago Cyclotron in 1952 [12].They observed a striking difference between the π+p and π-p total crosssections. The π-p cross section rose sharply from a few millibarns and came up toa peak of about 60 mb for an incident pion kinetic energy of 180 MeV. The π+pcross section behaved similarly except that for any given energy, its cross sectionwas about three times as large as that for π-p.[13]

$$pp \to d\pi^+$$
$$pn \to d\pi^0 \qquad 6$$

For a Deuterium is an "iso-singlet", i.e. it has $I = 0 \to |1,1>$

The Isospin states of the proton, neutron and pions are listed below. In terms of isospin states:

pp=|1/2,1/2>|1/2,+1/2> $d\pi^+$ =|0,0>|1,1>
pn=|1/2,1/2>|1/2,-1/2> $d\pi^0$ =|0,0>|1,0> 7

If considering the same techniques used to combine angular momentum in QM then we can go from 1/2 basis to the 1 basis. For pp, $d\pi^+$, and $d\pi^0$ there is only one way to combine the spin states:

pp=|1/2,1/2>|1/2,+1/2>=|1,1>
$d\pi^+$=|0,0>|1,1>=|1,1> 8
$d\pi^0$=|0,0>|1,0>=|1,0>

However, the pn state is tricky since it is a combination of |0,0> and |1,0>. The amount of each state is given by the Clebsch-Gordan coefficients ($1/\sqrt{2}$ in this cases).

$$|j_1,m_1\rangle|j_2,m_2\rangle = \sum_{j=|j_1-j_2|}^{j_1+j_2} C_{m,m_1,m_2}^{J,J_1,J_2}|j,m\rangle \ with \ m = m_1 + m_2 \qquad 9$$

$$|1/2,+1/2\rangle|1/2,-1/2\rangle = \frac{|0,0\rangle}{\sqrt{2}} + \frac{|1,0\rangle}{\sqrt{2}}$$

to calculate the ratio of scattering cross sections for these two reactions. Fermi's Golden Rules tells us that a cross section is proportional to the square of a matrix element:

$$\sigma \propto |\langle f|H|\mathrm{I}\rangle|^2 \qquad 10$$

withI=initial state, f =final state, H =Hamiltonian.If H conserves Isospin (strong interaction) then the initial and final states have to have the same I and I_3. Therefore assuming Isospin conservation then:

$|\langle d\pi^+|H|pp\rangle|^2 = |\langle 1,1||1,1\rangle|^2 = 1$
$|\langle d\pi^0|H|pn\rangle|^2 = |\langle 1,0|\ (1/2)(\ |0,0> + |1,0\rangle|^2 = 1/2$ 11

The ratio of cross section is expected to be:

$$\frac{\sigma_{pp} \to d\pi^+}{\sigma_{pn} \to d\pi^0} = \frac{|< d\pi^+|H|pp>|^2}{|< d\pi^0|H|pn>|^2} = \frac{2}{1} \qquad 12$$

This ratio is consistent with experimental measurement. The general construction of isospin amplitudes for π system taking into account Bose statistics has been given by Pairs [12] a long time ago but was only recently used for π physics [13].

Another case is the Isospin invariance which can be found in pion nucleon scattering. Consider the following two-body reactions of the eight reactions of the form $\pi N \to \pi' N'$ theonly three directly accessible experimentally are thosewith charged pion beams and proton targets: $\pi^{\pm} p$ elasticscattering, with amplitudes $f_{\pm}$, and $\pi^- p \to \pi^0 p$ charge-exchange(CEX) scattering, with amplitude f_{CEX} the Isospinconservation gives a relationship among these three amplitudes,the triangle identity

$$f_{CEX} = \frac{1}{\sqrt{2}}(f_+ - f_-) \qquad 13$$

The study of isospin-breaking effects is favored at lowenergies for the following reasons.

- Recent measurementsof pion-nucleon elastic [14] and charge-exchange [15]scattering for $T_\pi \leq 50$ MeV have yielded data of exceptionallyhigh quality. A new pionic atom measurementhas also been recently obtained [16].
- There is a minimumin each of the amplitudes in this energy range. Asmall value of any one of the amplitudes is useful sinceEq. (13) implies that if any of $(f_+, f_-, \sqrt{2} f_{CEX})$ is zerothe other two must have equal magnitudes.
- At these low energies the imaginary part of the amplitudes is verysmall.
- The s- and p-wave amplitudes suffice to describescattering and have a smooth and gentle energydependence.[17]

The existence of a minimum in the charge-exchange cross section at 0o is very important, since its position in energy can be determined much more precisely experimentally than the absolute magnitude of the cross section. Fitzgerald et al. [15] determined the energy of the minimum to be $T\pi = 45.1\ 6\ \pm 0.5$MeV.

State Isospin decomposition

π^+p $\quad |1.1\rangle\, |1/2.1/2\rangle = |3/2.3/2\rangle$

π^-p $\quad |1,\text{-}1\rangle |1/2,1/2\rangle = \sqrt{1/3}\,|3/2,-1/2\rangle - \sqrt{2/3}\,|1/2,-1/2\rangle$ 14

π^0n $\quad |1,0\rangle |1/2,\text{-}1/2\rangle = \sqrt{2/3}\,|3/2,-1/2\rangle + \sqrt{1/3}\,|1/2,-1/2\rangle$

If at a certain energy the scattering particles form a bound state with I=3/2 then only the I=3/2 components will contribute to the cross section, i.e.: or very small. Thus we have:

$\pi + \text{p} \rightarrow \pi + \text{p} = \langle 3/2,3/2|\, H\, |3/2,3/2\rangle$

$$\pi - \text{p} \rightarrow p - \pi = 1/3\left\langle \frac{3}{2}, -\frac{1}{2}\right| H \left| \frac{3}{2}, -\frac{1}{2}\right\rangle + 2/3\left\langle \frac{1}{2}, -\frac{1}{2}\right| H \left| \frac{1}{2}, -\frac{1}{2}\right\rangle = 1/3\left\langle \frac{3}{2}, -\frac{1}{2}\right| H \left| \frac{3}{2}, -\frac{1}{2}\right\rangle \quad 15$$

$$p - p \rightarrow \pi + p^0 n = \frac{1}{\sqrt{3}}\sqrt{\frac{2}{3}}\left\langle \frac{3}{2}, -\frac{1}{2}\right| H \left| \frac{3}{2}, -\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\frac{1}{\sqrt{3}}\left\langle \frac{1}{2}, -\frac{1}{2}\right| H \left| \frac{1}{2}, -\frac{1}{2}\right\rangle = \frac{\sqrt{2}}{3}\left\langle \frac{3}{2}, -\frac{1}{2}\right| H \left| \frac{3}{2}, -\frac{1}{2}\right\rangle$$

The cross sections depend on the square of the matrix element. If we assume that the strong interaction is independent of I_3 then we get the following relationships:

$$a)\ \sigma_{\pi^+ p \rightarrow \pi^+ p} : \sigma_{\pi^- p \rightarrow \pi^o n} : \sigma_{\pi^- p \rightarrow \pi^- p} = 9:2:1$$

$$b)\ \frac{\sigma_{\pi^- p \rightarrow \pi^o n}}{\sigma_{\pi^- p \rightarrow \pi^- p}} = 2 \quad 16$$

$c)$ For the total cross section: $\frac{\sigma_{\pi^+ p}}{\sigma_{\pi^- p}} = 3$

The three predictions are in good agreement with the data(Data from 1952 paper byFermi's group. They measuredthe cross section for $\pi^- p$ and $\pi^+ p$as a function of beam energy[18]) except equation (16c). Modern compilation of data from many experiments giving the cross section for $\pi^- p$ and $\pi^+ p$ as a function of the πp invariant mass.

II. METHODOLOGY

The isotopes of $^{56}Ni \rightarrow {}^{58}Ni, and\ {}^{206}Pb \rightarrow {}^{208}Pb$ and determined experimental energy of the T_π = 45.1 6 - 0.5 MeV which was used in this work, aFortran programmed KAP19 [19] was used in this research to calculates the differential and the total scattering cross-section for the scattering of a neutron with energy E ≤ 50 MeV by a target nucleus with mass number A ≥ 40.

Parameters used in the programmed are:

LMAX: Maximum angular momentum quantum number (LMAX=18)

IPI: Number of sub-intervals in the interval [0,π] (IPI=90)

IB: Number of integration steps for the calculation of the radial wave function (IB=500)

B: Upper integration limit for the calculation of the radial wave function in units of 1 fm (B=20.D0)

M: Mass of the neutron in units of 1 u (M=1.008665D0)

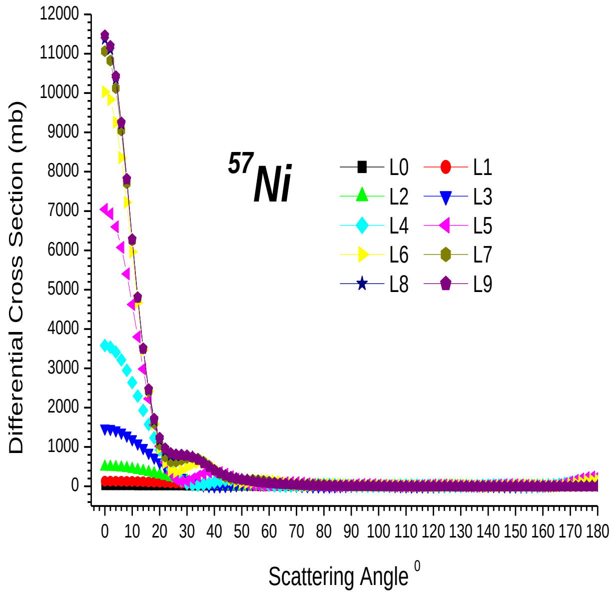
HBAR: PLANCK's constant divided by 2π in units of 1fmû(MeVu)

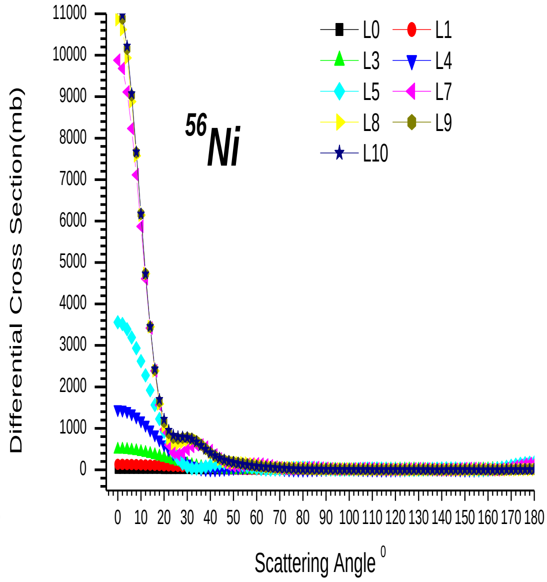
2.1 Input quantities

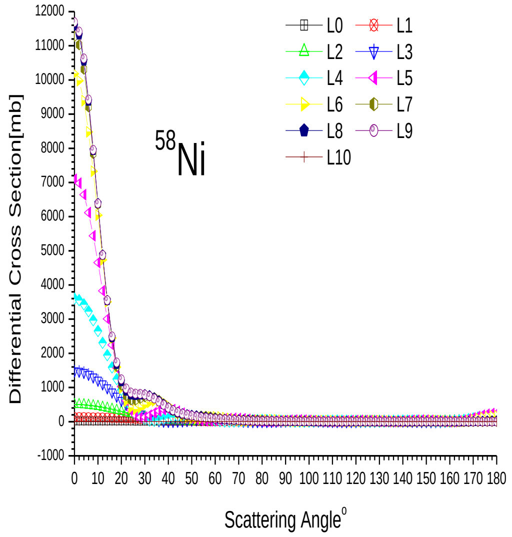
A: Mass number of the target nucleus (number of nucleons)
Z: Nuclear charge number of the target nucleus (number of protons)
E: Energy of the neutron in units of 1MeV
LP: Angular momentum quantum number; in the calculation of the interaction cross-section all partial waves up to and including LP are taken into account
Lead (Pb) with A=206,207,208, Z= 82 percentage abundance of 24.1, 22.1,52.4 and Nickel (Ni) with A= 56, 57, 58 Z = 28 percentage abundance of 59.930, 68.077, 26.223 [20]Angular momentum quantum number runs from $l = 0$ *to* 10.

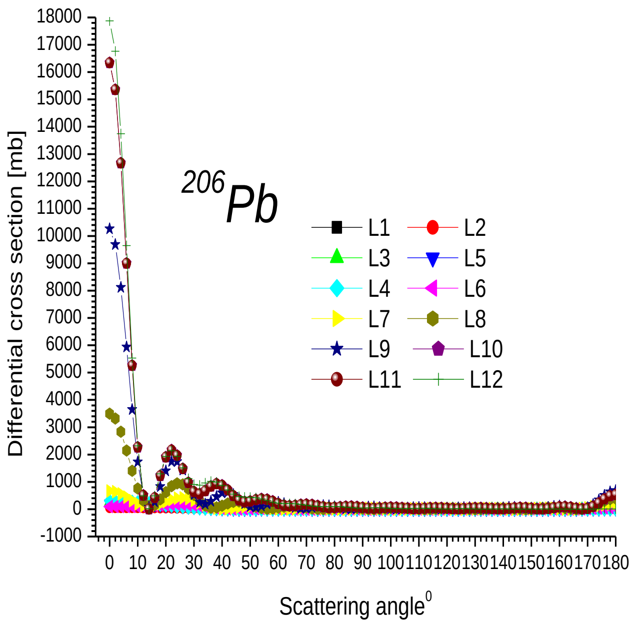
2.2 Output quantities
THEATA: Scattering angle
DSIGMA: Differential scattering cross-section in units of 1mb
SIGMA: Total scattering cross-section in units of 1mb

III. RESULTS









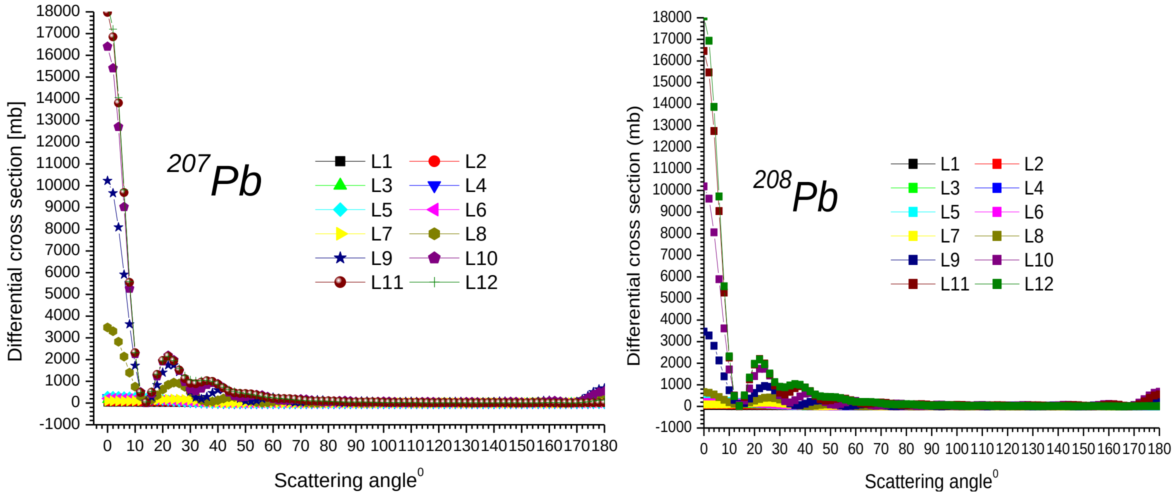


Fig1: plots of Differential cross section against scattering angle for angular momentum 1 to 10, for istopote of Lead and Nickel.

IV. DISCUSSION

Its shows the effect of quasistable/metastablefor both the system considered inNi58, Ni57 and Ni56 the highest pole is at 922.614mb, 721.697mb, 826.508mb at the scattering angle 31.58^0, 32^0, 32.03^0, and 29.05^{0}while for Pb208 Pb207, Pb206, the highest pole is at 2101.273mb, 2269.307mb, 2162.3303mb at the scattering angle 22.626^0, 21.974^0, 21.802^{0}Respectively. It was observed that in both Ni56 and Pb208 there is a phenomenon known as a `resonance particle,' which means there is a peak in the scattering cross section as a function of center of mass (CM)energy corresponding exactly to a short-lived particle ofcertain mass.

4.0 Conclusion

It's showed that the phase shift analysis had ambiguities and that the resonant hypothesis was not unique. It took another two years to settle fully the matter with many measurements and phase shift analyses.

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