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Circle-Criterion Observer design for nonlinear systems satisfying incremental quadratic constraints

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Abstract: In this paper, we design an Circle-Criterion observer for nonlinear systems satisfying incremental quadratic constraints. We make full use Circle-Criterion which can relax observer design condition. Nonlinear term is parameterized by a set of incremental multiplier matrices which can make state error converges to zero in exponential form. The result is illustrated by application to an example.
keywords: Circle-Criterion, incremental quadratic constraints, incremental multiplier matrices, exponential form

I. INTRODUCTION

The basic problem of the control system is how to view the real state of the system from the known input and output values. At present, the state of the system is obtained by using the method of observer or state estimation. Observer and state estimation can make the state estimation asymptotic to the real state of the system. Luenberger proposed typical observers which consist of system prototype with linear correction section which is composed of measurable output error. Then the asymptotic stability of the observer by establishing the dynamic stability of the state estimation error. But the design conditions of various observers are limited

We ameliorate the above problem. The contributions of this paper include the following:(i)Nonlinear model used in this paper which the nonlinear/time varying which satisfied incrementalquadratic constraints[1] lie in the input.(ii) We introduce Circle-Criterion which the linear output error which is introduced into nonlinear term.This can relax the observer design conditions[2,3,4]. The convergence effect of state error estimation is better. (iii)We the theory of circle criterion and the incremental quadratic constraints[10] to vehicle state estimation

The rest of this paper is organized as followed. In section 2,we propose a description of the systems under consideration.We use incremental multiplier to generalize the nonlinear part of the system.In section 3,the Circle-Criterionobserver is presented.In section 4, the article gives a two wheelmotorcycle model with two degrees of freedom a simulation.Conclusions are given in section 5.The acknowledge is givenin section 6.

II. SYSTEM DESCRIPTION AND INCREMENAL QUADRATIC CONSTRAINTS

In this paper, we consider a class of nonlinear system categories as followed

$$\begin{aligned} \dot{x} &= Ax + Bu + G_{\Phi}\Phi(x) \\ y &= Cx \end{aligned} \qquad (1)$$

where $x \in R^{n_x}$ is the state vector, $u \in R^{n_u}$ which is the known input. $y \in R^{n_y}$ is the vector of measurable output. The nonlinear vector valued function Φ represent the system nonlinearities.

The A, B, C, G_{Φ} have continuous and proper size.

First,we started by imposing a constraint on the nonlinearities of system(1). The Φ should satisfy incremental quadratic constraints(δQC)(10). Let M denote the set of symmetric matrices $M \in \mathrm{M}$ which are incremental matrices, when it satisfies incremental quadratic constraints(δQC).

$$\begin{bmatrix} \delta v \\ \delta\Phi \end{bmatrix}^T M \begin{bmatrix} \delta v \\ \delta\Phi \end{bmatrix} > 0 \tag{2}$$

where $\delta v = \hat{v} - v$ (3)
and

$$\delta\Phi = \Phi(\hat{v}) - \Phi(v) \tag{4}$$

In order to illustrate the limiting condition, we will give an example as followed

III. EXPONENTIAL OBSERVER DESIGN

In this section, we consider the observer design for the system(1) under consideration which Φ meet the condition(2)(δQC). Inspired by[1,2,3,4,10], we construct an observer in the form of

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + G_\Phi \Phi(\hat{x} + L_1(\hat{y} - y)) + L(\hat{y} - y) \\ y &= Cx \end{aligned} \tag{5}$$

where $\hat{x} \in R^{n_x}$ which is the estimate of the state vector x. The gain matrices L_1 and L are the output injection gain which are constant matrices of appropriate size. The $L_1(C_1\hat{x} - y)$ can relax the feasibility for observer design[4,5,6].

The paper's task is to find the appropriate gain matrices L_1 and L. These gain matrices can make the state error $e = \hat{x} - x$ converge to zero quickly. The state error is governed by

$$\dot{e} = \dot{\hat{x}} - \dot{x} = Ae + G_\Phi \delta\Phi + L(C\hat{x} - Cx) = Ae + LCe + G_\Phi \delta\Phi \tag{6}$$

According to (2),(3) and (4), the Φ can be transformed into the following inequality

$$\begin{bmatrix} e \\ \delta\Phi \end{bmatrix}^T \begin{bmatrix} I + L_1 C & 0 \\ 0 & I \end{bmatrix}^T M \begin{bmatrix} I + L_1 C & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} e \\ \delta\Phi \end{bmatrix} \geq 0 \tag{7}$$

proof:

$$\begin{bmatrix}\delta v\\ \delta\Phi\end{bmatrix}^T M\begin{bmatrix}\delta v\\ \delta\Phi\end{bmatrix}=\begin{bmatrix}\hat{x}+L_1(C\hat{x}-Cx)-x\\ \delta\Phi\end{bmatrix}^T M\begin{bmatrix}\hat{x}+L_1(C\hat{x}-Cx)-x\\ \delta\Phi\end{bmatrix}$$
$$=\begin{bmatrix}e+L_1Ce\\ \delta\Phi\end{bmatrix}^T M\begin{bmatrix}e+L_1Ce\\ \delta\Phi\end{bmatrix}$$
$$=\begin{bmatrix}e\\ \delta\Phi\end{bmatrix}^T\begin{bmatrix}I+L_1C & 0\\ 0 & I\end{bmatrix}^T M\begin{bmatrix}I+L_1C & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}e\\ \delta\Phi\end{bmatrix}$$
$$=\begin{bmatrix}e\\ \delta\Phi\end{bmatrix}^T\phi^T M\phi\begin{bmatrix}e\\ \delta\Phi\end{bmatrix}$$

where $\phi=\begin{bmatrix}I+L_1C & 0\\ 0 & I\end{bmatrix}$ (8)

Theorem 1 Suppose that the system(1) satisfies the δQC (2)condition. The observer has the form of(7).If the matrices and with appropriate dimensions, and the scalar such that the following matrix inequality is satisfied:

$$\begin{bmatrix}\Pi+\alpha I & PG_\Phi\\ G_\Phi{}^T P & 0\end{bmatrix}+\phi^T M\phi\le 0 \tag{9}$$

where ϕ and Π are given by (11), (12) and (14) respectively.

$$\Pi=A^TP+PA+C^TL^TP+PLC \tag{10}$$

then, the state estimation error converges to zero in exponential form[11]; that is,

$$\|e(t)\|\le\sigma\,\|e(t_0)\|\,e^{-\gamma(t-t_0)} \tag{11}$$

where $\sigma=\sqrt{\lambda_{\max}(P)/\lambda_{\min}(P)}$ *and* $\gamma=\dfrac{\alpha}{2\lambda_{\max}(P)}$ *are positive scalars.*

*Proof :*For the state estimation error system(6), we consider the Lyapunov function $V(e)=e^TPe$

we have

$$\begin{aligned}\dot{V}&=\dot{e}^TPe+e^TP\dot{e}=(Ae+G_\Phi\delta\Phi+LCe)^TPe+e^TP(Ae+G_\Phi\delta\Phi+LCe)\\ &=(e^TA^T+\delta\Phi^TG_\Phi{}^T+e^TC^TL^T)Pe+e^TP(Ae+G_\Phi\delta\Phi+LCe)\\ &=e^TA^TPe+\delta\Phi^TG_\Phi{}^TPe+e^TC^TL^TPe+e^TPAe+e^TPG_\Phi\delta\Phi+e^TPLCe\\ &=\begin{bmatrix}e\\ \delta\Phi\end{bmatrix}^T\begin{bmatrix}A^TP+PA+C^TL^TP+PLC & PG_\Phi\\ G_\Phi{}^TP & 0\end{bmatrix}\begin{bmatrix}e\\ \delta\Phi\end{bmatrix}\end{aligned} \tag{12}$$

The Φ are satisfy incremental quadratic constraints(2),
So we have

$$\begin{bmatrix} e \\ \delta\Phi \end{bmatrix}^T \begin{bmatrix} I+L_1C & 0 \\ 0 & I \end{bmatrix}^T M \begin{bmatrix} I+L_1C & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} e \\ \delta\Phi \end{bmatrix} > 0$$

This expression implies that pre- and post-multiplying both sides of inequality(6) $\begin{bmatrix} e^T & \delta\Phi^T \end{bmatrix}$ and its transpose results as followed.

$$\begin{aligned} &\begin{bmatrix} e \\ \delta\Phi \end{bmatrix}^T \begin{bmatrix} \Pi+\alpha I & P \\ P & 0 \end{bmatrix} \begin{bmatrix} e \\ \delta\Phi \end{bmatrix} + \begin{bmatrix} e \\ \delta\Phi \end{bmatrix}^T \phi^T M \phi \begin{bmatrix} e \\ \delta\Phi \end{bmatrix} \\ &= \begin{bmatrix} e \\ \delta\Phi \end{bmatrix}^T \begin{bmatrix} A^TP+PA+C^TL^TP+PLC & PG_\Phi \\ G_\Phi{}^T P & 0 \end{bmatrix} \begin{bmatrix} e \\ \delta\Phi \end{bmatrix} + \alpha \left\| e(t) \right\|^2 + \\ &\begin{bmatrix} e \\ \delta\Phi \end{bmatrix}^T \phi^T M \phi \begin{bmatrix} e \\ \delta\Phi \end{bmatrix} \\ &= \dot{V} + \alpha \left\| e(t) \right\|^2 + \begin{bmatrix} e \\ \delta\Phi \end{bmatrix}^T \phi^T M \phi \begin{bmatrix} e \\ \delta\Phi \end{bmatrix} \le 0 \end{aligned} \tag{13}$$

where ϕ is given by(8).

Through above inequality, we can get the result as followed that

$$\dot{V} \le -\alpha \left\| e(t) \right\|^2 \tag{14}$$

The above inequality means that the state estimation error(8)converges exponentially to zero. The above process proves the correctness of the theory.

Remark 1 The(9) is not linear in the variables P, L_1, L, α and involves the quadratic term $\phi^T M \phi$. (8) will become an LMI in the variables P, PL, α, if we partition M [11] as

$$M = k \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \tag{15}$$

where $k > 0, M_{11} = M_{11}{}^T, M_{12} = M_{21}{}^T, M_{22} = M_{22}{}^T$. By letting $R = PL$, the resulting o bserver gain matrices $L = P^{-1}R$. Then the (9)becomes

$$\begin{bmatrix} \square+\alpha I & M_{12}(I+C^TL_1{}^T)+P \\ M_{12}^T(I+L_1C)+P & M_{22} \end{bmatrix} \le 0 \tag{16}$$

proof:

$$\begin{bmatrix} A^TP+PA+C^TL^TP+PLC+\alpha I & PG_\Phi \\ G_\Phi{}^TP & 0 \end{bmatrix}+\begin{bmatrix} I+L_1C & 0 \\ 0 & I \end{bmatrix}^T\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix}\begin{bmatrix} I+L_1C & 0 \\ 0 & I \end{bmatrix}$$

$$=\begin{bmatrix} A^TP+PA+C^TL^TP+PLC+\alpha I & PG_\Phi \\ G_\Phi{}^TP & 0 \end{bmatrix}+\begin{bmatrix} I+C^TL_1^T & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix}\begin{bmatrix} I+L_1C & 0 \\ 0 & I \end{bmatrix}$$

$$=\begin{bmatrix} A^TP+PA+C^TL^TP+PLC+\alpha I & PG_\Phi \\ G_\Phi{}^TP & 0 \end{bmatrix}+$$

$$\begin{bmatrix} M_{11}+C^TL_1{}^TM_{11} & M_{12}+C^TL_1{}^TM_{12} \\ M_{12}^T & M_{22} \end{bmatrix}\begin{bmatrix} I+L_1C & 0 \\ 0 & I \end{bmatrix}$$

$$=\begin{bmatrix} A^TP+PA+C^TL^TP+PLC+\alpha I & PG_\Phi \\ G_\Phi{}^TP & 0 \end{bmatrix}$$

$$+\begin{bmatrix} M_{11}+C^TL_1{}^TM_{11}+M_{11}L_1C+C^TL_1{}^TM_{11}L_1C & M_{12}+C^TL_1{}^TM_{12} \\ M_{12}^T(I+L_1C) & M_{22} \end{bmatrix}$$

$$=\begin{bmatrix} \square+\alpha I & M_{12}+C^TL_1{}^TM_{12}+PG_\Phi \\ M_{12}^T(I+L_1C)+G_\Phi{}^TP & M_{22} \end{bmatrix}\le 0$$

$$where\ \square = A^TP+PA+C^TR^T+RC+M_{11}+C^TL_1{}^TM_{11}+M_{11}L_1C+C^TL_1{}^TM_{11}L_1C, R=PL$$

Hence, When the(9) has a feasible solution, a necessary condition is $M_{22}\le 0$. In addition, when $M_{11}=0$, (16) is an LMI in the variables P,α,R,L_1. Then by using the Shur complement lemma, (16) becomes an LMI in the following form

$$\begin{bmatrix} \theta+\alpha I & (I+C^TL_1{}^T)M_{12}+PG_\Phi & C^TL_1{}^TM_{11} \\ M_{12}^T(I+L_1C)+G_\Phi{}^TP & M_{22} & 0 \\ M_{11}L_1C & 0 & -M_{11} \end{bmatrix}\le 0 \qquad (17)$$

where $\theta = A^TP+PA+C^TR^T+RC+M_{11}+C^TL_1{}^TM_{11}+M_{11}L_1C$

proof:

$$\begin{bmatrix} \square+\alpha I & (I+C^TL_1{}^T)M_{12}+PG_\Phi \\ M_{12}^T(I+L_1C)+G_\Phi{}^TP & M_{22} \end{bmatrix}$$

$$=\begin{bmatrix} \theta+\alpha I+C^TL_1{}^TM_{11}L_1C & (I+C^TL_1{}^T)M_{12}+PG_\Phi \\ M_{12}^T(I+L_1C)+G_\Phi{}^TP & M_{22} \end{bmatrix}$$

The next we introduce Shur complements lemma.

$$\begin{bmatrix} \square + \alpha I & (I + C^T L_1^{\ T}) M_{12} + P G_\Phi \\ M_{12}^T (I + L_1 C) + G_\Phi^{\ T} P & M_{22} \end{bmatrix}$$

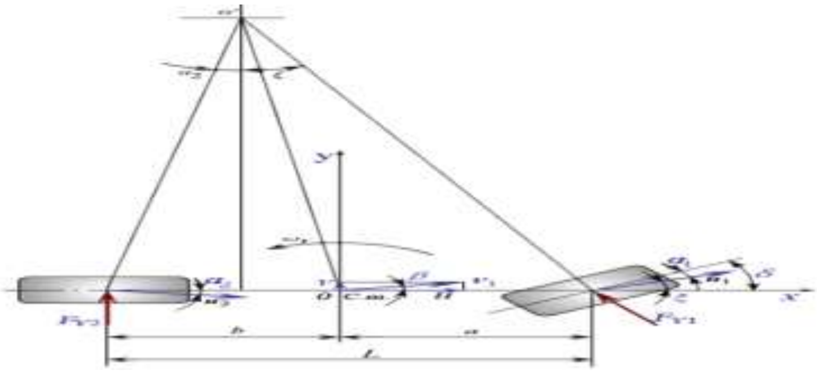
$$= \begin{bmatrix} \theta + \alpha I + C^T L_1^{\ T} M_{11} L_1 C & (I + C^T L_1^{\ T}) M_{12} + P G_\Phi \\ M_{12}^T (I + L_1 C) + G_\Phi^{\ T} P & M_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \theta + \alpha I & (I + C^T L_1^{\ T}) M_{12} + P G_\Phi & C^T L_1^{\ T} M_{11} \\ M_{12}^T (I + L_1 C) + G_\Phi^{\ T} P & M_{22} & 0 \\ M_{11} L_1 C & 0 & -M_{11} \end{bmatrix}$$

IV. ILLUSTRATIVE EXAMPLE

In this section, we will explain the application of the theoretical results to the yaw rate and sideslip angle estimation problem. It is a challenging task in vehicle initiative security system. As depicted in Fig.1 as followed.

the vehicle is simplified as a two wheel motorcycle modelwith two degrees of freedom.



$$\begin{aligned} \dot{w}_r &= \frac{a^2 k_1 + b^2 k_2}{I_z u} w_r + \frac{a k_1 - b k_2}{I_z} \beta - \frac{a k_1}{I_z} \delta + w_r^{\ 3} \\ \dot{\beta} &= (\frac{a k_1 - b k_2}{m u^2} - 1) w_r + \frac{k_1 + k_2}{m u} \beta - \frac{k_1}{m u} \delta + \beta^3 \end{aligned} \qquad (18)$$

where $m = 1818.2kg, a = 1.463m, b = 1.585m, k_1 = -62618N/rad, k_2 = -110185N/rad$

$I_z = 3889kg/m^2$ and $\delta = 20m/s$.

To design an observer for $\begin{bmatrix} w_r & \beta \end{bmatrix}$ from the above two degree of freedom vehicle model , Φ is nondecreasing . Rewrite(18) as in (1) with

$$A = \begin{bmatrix} -4.7266 & -3.0632 \\ -0.2932 & -4.2524 \end{bmatrix}, B = \begin{bmatrix} 23.5562 \\ 1.5409 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, G_\Phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and $\Phi(w_r,\beta)=\begin{bmatrix} w_r^{\,3} \\ \beta^3 \end{bmatrix}$

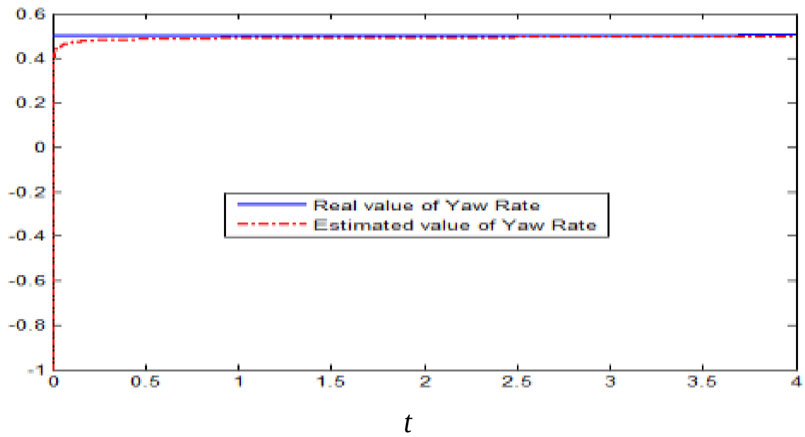


Fig.2. Vehicle yaw rate w_r and its observer estimate $\hat{w}_r$

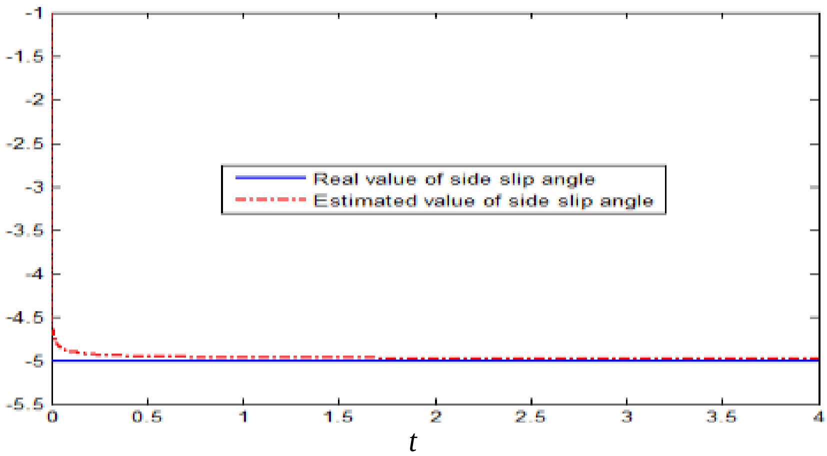


Fig.3. Vehicle Sideslip Angle β and its observer estimate $\hat{\beta}$

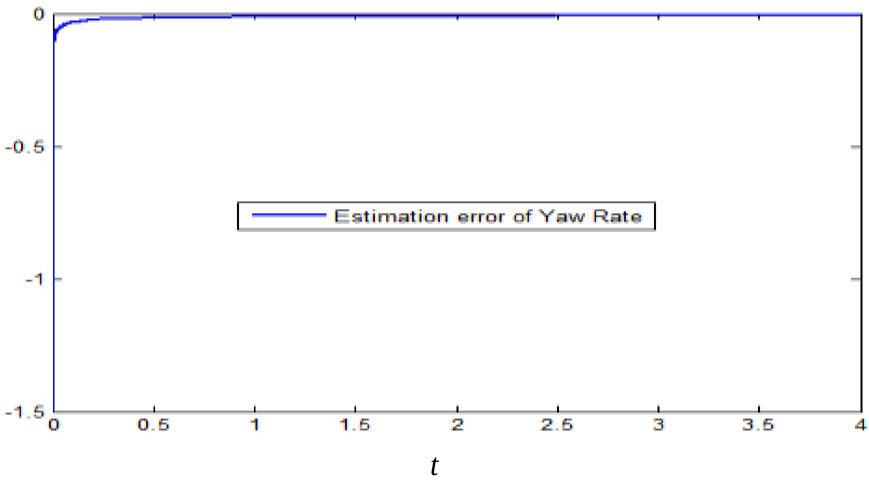


Fig.4. the estimation error of w_r

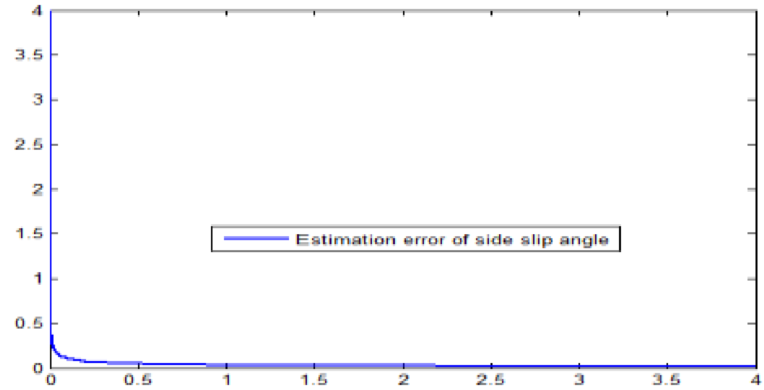


Fig.5. the estimation error of β

To further relax the feasibility conditions , we introduce Circle-Criterion method. The LMI(17) is feasible and a solution is

$$P=\begin{bmatrix} 8720.0571 & -96.352949 \\ -96.352949 & 8200.1496 \end{bmatrix}$$

$$L=\begin{bmatrix} 4.2239129 & 0.0496316 \\ 3.2563841 & 3.7185515 \end{bmatrix}$$

$$L1=\begin{bmatrix} 8.6352032 & -820.01496 \\ -1744.0114 & 18.270498 \end{bmatrix}$$

From Theorem 1, observer (5) ensures exponential convergence of the state estimates $\hat{w}_r$ and $\hat{\beta}$, As shown in Figure with simulations in Fig.2 , Fig.3, Fig.4 and Fig .5.

V. CONCLUTIONSC

This paper have considered the problem of state estimation for systems whose nonlinear terms satisfy an incremental quadrtic inequlity that is parameterized by a set of incremental multiplier matrices. We followed the method of circle-criterion observer design which is good ways to relax the conditions of observer design. The observer design involves solving LMI for the observer gains. We have used the method of circle-criterion observer design and incremental quadratic constraints conditions in the vehicle state estimation. The simulation results prove the correctness of our theory. It is hoped that these results will reinvigorate the search for new structure nonlinear observer design.

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