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Weak Signal Detection Research by Fractional Order Chaotic System And Non-Detection Area Removed Algorithm

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Abstract : After introducing fractional damping into improved Duffing System, a weak signal detecting system based on fractional Duffing was built in Simulink platform. Through simulation and analysis, the conclusion that fractional order Duffing system was better than integer order Duffing system in anti-noise performance under the same experimental conditions was proved. By analyzing the improved fractional Duffing equation, the Non-detection area was existed when the system was detecting the same frequency weak signal. This paper provided a method of eliminating the Non-detection area, and verified its feasibility by experimental result.
Keywords: Fractional calculus, Chaos, Duffing System ,Non-detection area

I. INTRODUCTION

In recent years, chaos detection had become an important research field of nonlinear science. Chaos detection refered to use the characteristics of chaotic system which were the strong immunity of noise and the sensitivity of the initial value to make the essential change of phase diagram. The research history of the fractional calculus was more than 300 years, but until recently, it was realized that the fractional calculus has a better effect on describing the complex problem. The combination of the two provided a new direction for the research of weak signal detection.

The classic Duffing oscillator can be regarded as a describtion of nonlinear vibration produced by the large amplitude oscillation of pendulum containing viscous damping .At present, Duffing oscillator to achieve effective detection of weak sinusoidal signal, chirp signal, the spectrum of ship. In the classic Duffing equation, Li Yue transformed the nonlinear, so that the system has a better effect of weak signal detection.

In this paper, we discuss the improvement of the classical Duffing oscillator chaotic system and the introduction of the fractional order calculus, and get the advantage of the new detection system through the comparative analysis. The non detection zone of the system is derived, and the effectiveness of the method is proved by experiments.

II. CONSTRUCTION OF IMPROVED FRACTIONAL ORDER DUFFING SYSTEM

The classical Duffing-Homes equation is

$$\ddot{x} + k\dot{x} - x + x^3 = \gamma\cos(\omega t) \tag{1}$$

If the recovery force item is changed from ($-x + x^3$)to($-x^3 + x^5$), the formula (1) becomes

$$\ddot{x} + k\dot{x} - x^3 + x^5 = \gamma\cos(\omega t) \tag{2}$$

In this equation ,the damping ratio is k ,the system's periodic driving force is $\gamma\cos(\omega t)$,accompanied by the change in γ ,The system exhibits the state of periodic oscillation, chaos and large period. Remember the critical value between chaos and large period state is γ_{d} ,the principle of weak signal detection by using fractional order Duffing oscillator is: γ_{d} was set to the critical value of the system, when the weak signal is added, the total driving force of the system will be greater than γ_{d} .so as to realize the detection of weak signal according to the change of system state.

To facilitate the discussion, the formula (2) can be written as

$$D^2 + kDx - x^3 + x^5 = \gamma\cos(\omega t) \tag{3}$$

D represents the first order differential operator, D^2 represents the second order differential operator. Convert to the easy simulation model

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = \gamma \cos(\omega \mathrm{t}) - ky + x^3 - x^5 \end{cases} \tag{4}$$

By introducing the fractional order, the first order differential operator in the formula (3) is changed into the fractional order

$$D^2 + kD^{\mathrm{q}} x - x^3 + x^5 = \gamma \cos(\omega t) \tag{5}$$

among $0 < \mathrm{q} < 1$

According to the fractional calculus's property of superposition and Commutative law,the formula (5) can be converted to a set of equations with fractional order

$$\begin{cases} \dfrac{d^q x}{dt^q} = y \\ \dfrac{d^{1-q} y}{dt^{1-q}} = z \\ \dfrac{dz}{dt} = \gamma \cos(\omega \mathrm{t}) - ky + x^3 - x^5 \end{cases} \tag{6}$$

III. SYSTEM SIMULATION ANALYSIS

Although the universality of integer order approximation method is relatively high, but the conversion is complex, if the equation contains high order terms or the order was increased will increase the error rate of mamual conversion and the computational burden of computer. Therefore, the Simulink software platform was introduced to simulate the fractional order equation, which no longer need the complicated calculation and easy for hardware implementation.

The key to the simulation of fractional order equation is to simulate the fractional operator. In the simulation, we use the transfer function to carry out. Transfer function can be realized by transforming the transfer function into the state space. The State-space module can be used to transfer function conversion.

Simulation analysis of integer order chaotic system

Improved integer order Duffing system equations, such as formula (4), take k=0.5, $\omega = 1.0 \mathrm{rad/s}$ .Initial value $x = 0$, $x' = 0$ .The simulation points are 50000.

Firstly, the integer order Duffing system is adjusted to the critical chaotic state. here $\gamma = 0.7250$,When γ is 0.7250 and 0.7251, the system phase diagram is shown in Fig.1 and Fig.2.

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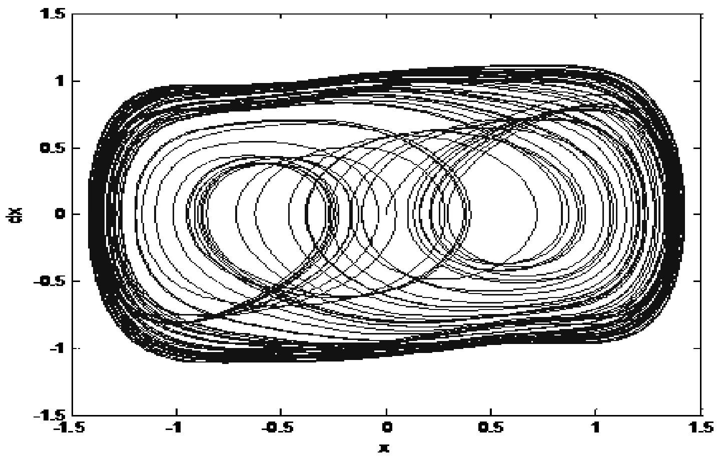

Fig.1 Phase diagram of chaotic critical state of integer order Duffing system

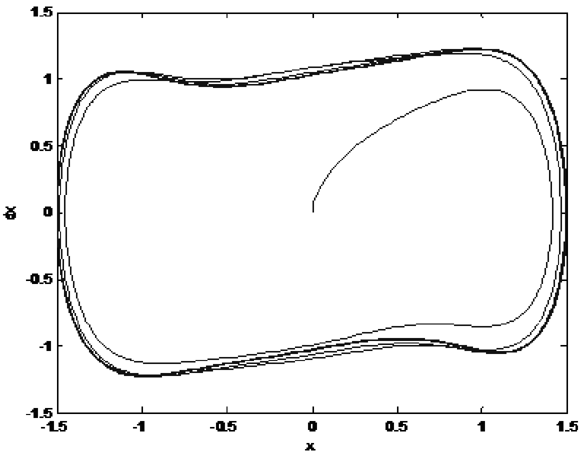

Fig.2 Phase diagram of integer order Duffing system with noise and input signal

2 Simulation analysis of fractional order chaotic system

Improved fractional order Duffing system equations, such as formula (6),take fractional order q=0.5,c=0.5, $\omega = 1.0\text{rad}/\text{s}$ .Initial value $x = 0$, $x' = 0$ .Adjusting the driving force γ to make the system in chaos critical state. Measured critical value is 4.1375.When γ is 0.7250 and 0.7251, the system phase diagram and power spectrum diagram are shown in Fig.3、Fig.4 and Fig.5、Fig.6.

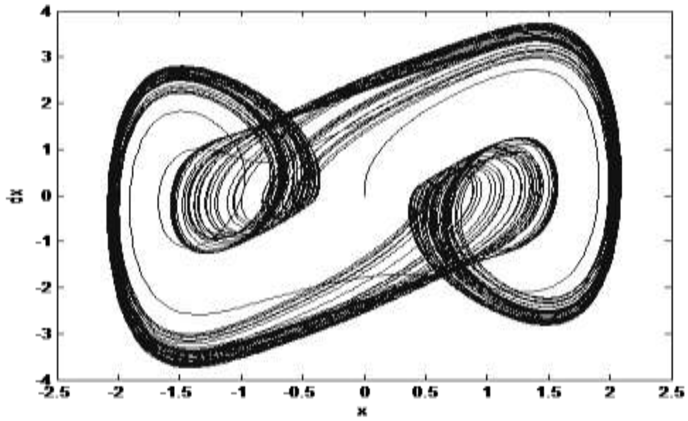

Fig.3 Phase diagram of the noise variance is 10^{-2} and γ is 4.1375

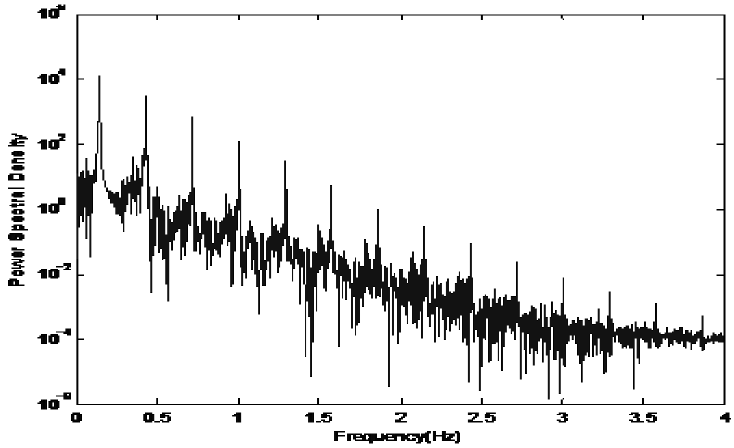

Fig.4 Power spectrum of the noise variance is 10^{-2} and γ is 4.1375

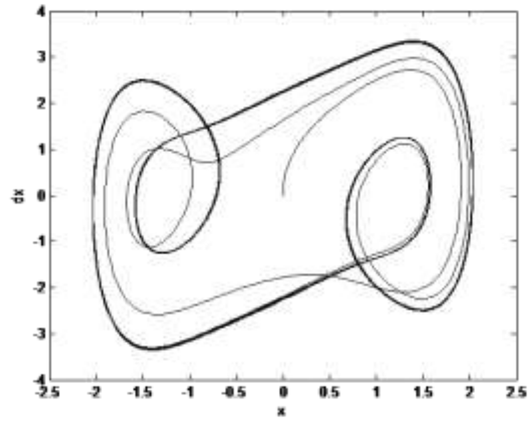

Fig.5 Phase diagram of the noise variance is 10^{-4} and γ is 4.1376

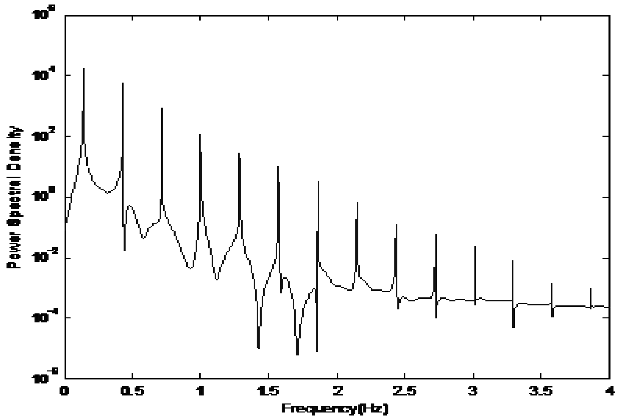

Fig.6 Power spectrum of the noise variance is 10^{-4} and γ is 4.1376

3 Comparative analysis of the system performance under different orders

In addition, the Duffing system with other orders and integer order Duffing system are selected as the control group.Also select c=0.5, $\omega = 1.0\text{rad/s}$,Comparison of detection accuracy is shown in Table.1.

Order	SNR（dB）
1	-21.46
0.3	-39.03
0.4	-52.04
0.5	-63.01
0.6	-63.01
0.7	-46.02

Table.1 Detection signal to noise ratio of different order systems with $\omega = 1.0\text{rad/s}$

From the Table.1 shows that in the case of the same parameters, the fractional order Duffing system has obvious advantages compared with the integer order Duffing oscillator system in terms of detection accuracy.

IV. NON DETECTION ZONE AND REMOVAL METHOD

1 Theoretical derivation of non-detection zone

Improved fractional order Duffing formula is

$$\ddot{x} + k\dot{x} - x^3 + x^5 = \gamma\cos(\omega t) \tag{7}$$

Set the signal to be measured as $s(t) = A\cos(\omega_x t + \varphi) = A\cos((\omega + \Delta\omega)t + \varphi)$ (among $\omega_x = \omega + \Delta\omega$),Because the Duffing system has strong noise immunity, it is not considered the influence of noise. The total driving force of the equation (7) is

$$\gamma_d \cos(\omega t) + s(t) = \gamma_d \cos(\omega t) + A\cos(\omega + \Delta\omega) + \varphi) = \gamma(t)\cos(\omega t + \theta(t)) \quad (8)$$

Among $\gamma(t)$ is the amplitude of total driving force, $\theta(t)$ is the phase. expressions are

$$\gamma(t) = \sqrt{\gamma_d^2 + 2\gamma_d A\cos(\Delta\omega t + \varphi) + A^2} \quad (9)$$

$$\theta(t) = \frac{A\sin(\Delta\omega t + \varphi)}{\gamma_d + A\cos(\Delta\omega t + \varphi)} \quad (10)$$

The amplitude of the weak signal to be measured A is usually far less than γ_d ,so $\theta(t)$ can be approximately equal to 0.The application of this system for weak signal detection, the main is to discuss the relationship between $\gamma(t)$ and γ_d ,by judging the state of system to achieve weak signal detection.

When the weak signal to be measured is at the same frequency as the driving force of the system, namely $\Delta\omega = 0$,the amplitude of total driving force is

$$\gamma(t) = \sqrt{\gamma_d^2 + 2\gamma_d A\cos\varphi + A^2} \quad (11)$$

When $\gamma(t) > \gamma_d$, $\varphi \in (0, \pi - \arccos(A/(2F_d))) \cup (\pi + \arccos(A/(2F_d)), 2\pi)$ can be obtained by derivation, because $A << \gamma_d$,it can be approximated as $\varphi \in (0, \pi/2) \cup (3\pi/2, 2\pi)$,At this point, the system can be changed from chaos to large period, and the weak signal to be measured can be detected by the change of system state. And when $\gamma(t) \le \gamma_d$, $\varphi \in [\pi - \arccos(A/(2F_d)), \pi + \arccos(A/(2F_d))]$,because $A << \gamma_d$,it can be approximated as $\varphi \in (\pi/2, 3\pi/2)$ .At this time the system is still in chaos state, which means that the system does not change the state even joined the weak signal to be detected, which means that the weak signal to be detected falls into the Non detection zone of the system.

2 Removal method of Non detection zone

From the analysis of upper section, the phase range of detection zone and non-detection zone can be approximated as $\varphi \in (0, \pi/2) \cup (3\pi/2, 2\pi)$ and $\varphi \in (\pi/2, 3\pi/2)$,The following conclusion can be obtained: if the phase of the weak signal to be detected φ is in the non-detection zone, then $\pi - \varphi$ is in the detection zone. To sum up, the removal method of non-detection zone can be summarized as follows: In the model based on the formula (6), if φ is in the non-detection zone, then make the initial phase of system driving force move π ,If the system changes from chaos to large periodic state, it is indicated that the non-detection zone is removed successfully, and the detection of weak signal to be detected is realized. If the system is still in a state of chaos, then the weak signal to be detected does not exist.

In order to remove the non-detection zone, the Lyapunov index is introduced here. Lyapunov index is an important index for judging the state of chaotic system. The Lyapunov exponent is less than zero, corresponding to the stable fixed point and periodic motion, and the Lyapunov exponent is greater than zero, corresponding to the chaotic state of the system. and the greater the index, the more obvious the chaotic characteristics, the higher the degree of chaos.

The flow chart of the non-detection zone removal algorithm is as Fig.7

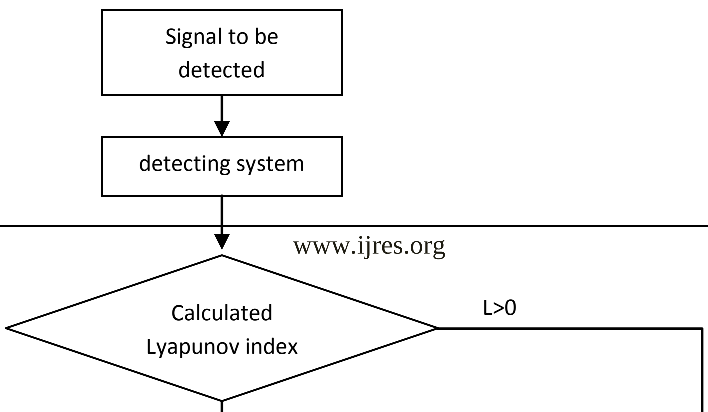

Fig.7 The flow chart of the non-detection zone removal algorithm

First, the signal to be detected is transmitted into the detection system, and the Lyapunov index is calculated. If the Lyapunov index is less than zero, the system changes from chaos to periodic state, and the detection of weak signal is realized. If Lyapunov index is greater than zero, move the initial phase of driving force(),then calculated Lyapunov index again. If the Lyapunov index is less than zero, the detection of weak signal is realized, which illustrates that successfully moves the weak signal to be detected from the non-detection zone to the detection zone, If the Lyapunov index is still greater than zero, it can be explained that there is no weak signal to be detected.

3 Simulation Experiment

When the initial phase of the detection system is 0, the critical value is adjusted to the critical state, and the critical value $\gamma_d = 4.1375$, The system phase diagram is shown in Fig.8

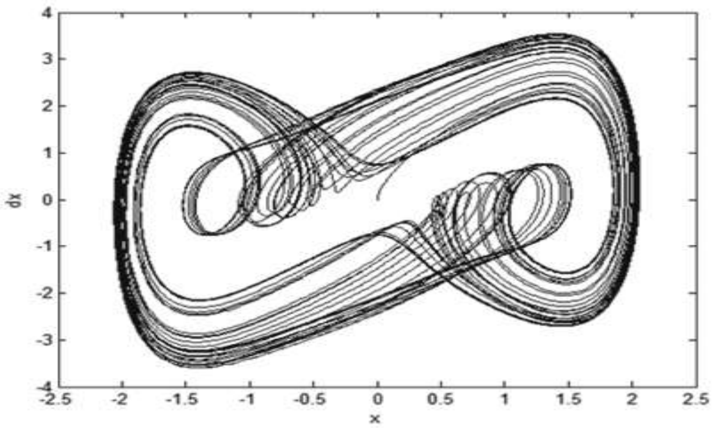

Fig.8 The system phase diagram of the critical chaotic state with initial phase is 0

Adding the weak signal of 0.0001 amplitude, $\omega = 1$ and $\varphi = 1.1\pi$ which is in the non-detection zone. The system phase diagram is shown in Fig.9.

From Fig.10, the maximum Lyapunov index is positive (at least another Lyapunov index is negative), which means the system is still in chaos state.

The Poincare section of Fig.11 shows the dense points with a hierarchical structure, and it can also illustrate the chaotic state of the system. That is to say, after adding the weak signal to be detected, there is no change in the system state.

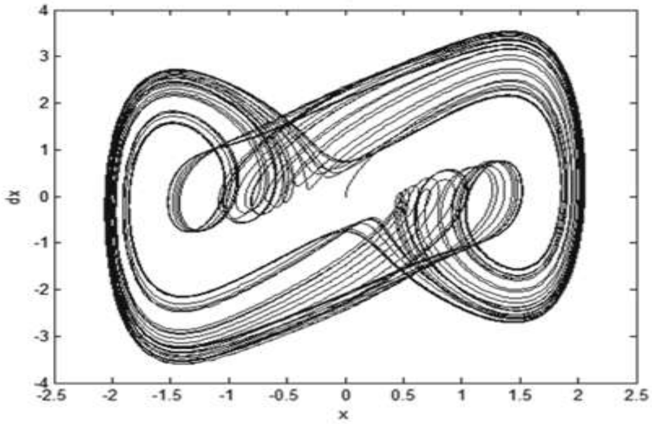

Fig.9 The phase diagram of the system added the signal to be detected.

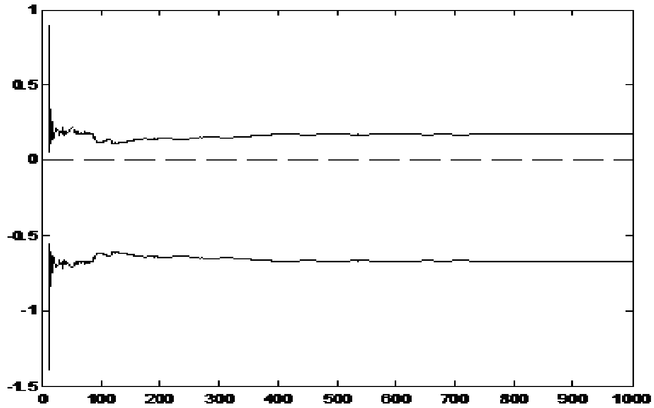

Fig.10 Lyapunov index

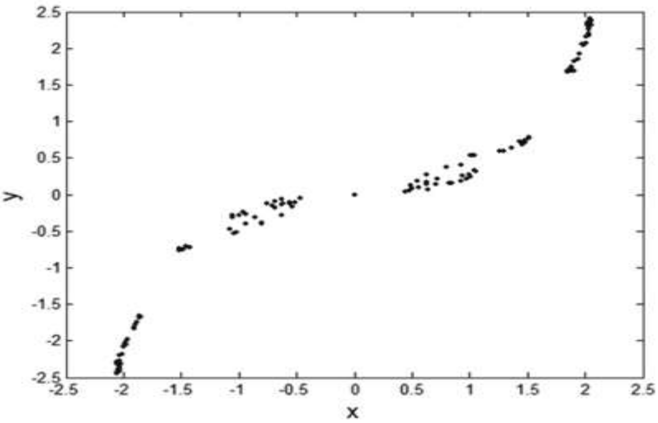

Fig.11 Poincare section

To move the phase π by using the method of non-detection zone introduced in the previous section.

After moving the phase, the state of the system is changed to the large periodic state, as shown in Fig.12.And as shown in Fig.13, the largest Lyapunov index is negative, indicating that the system is in the large periodic state.And the discrete finite points in Poincare section of Fig.14 can also indicate that the sa state of large cycle.iodic state.

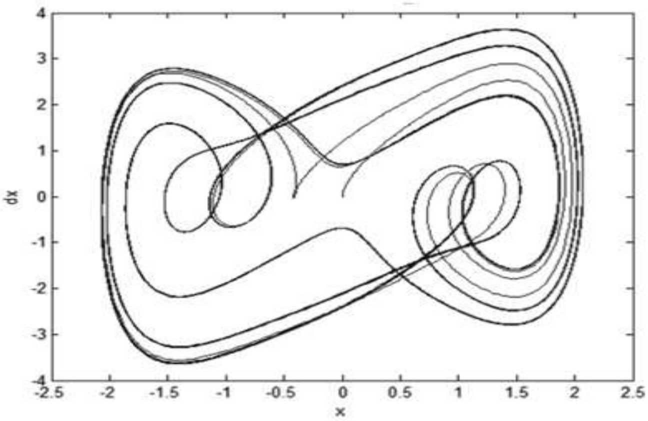

Fig.12 system phase diagram after Phase shift

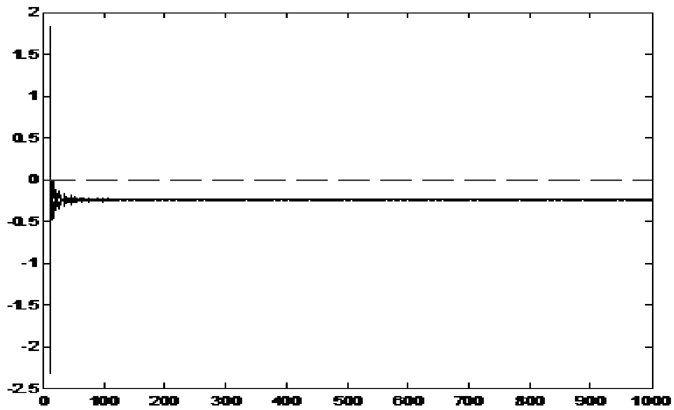

Fig.13 Lyapunov index

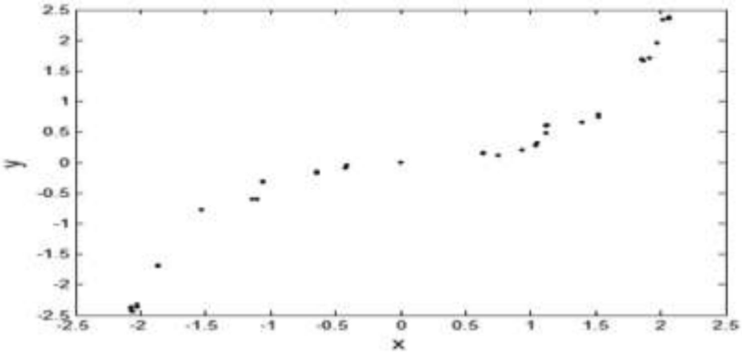

Fig.14 Poincare section

The maximum variance of the added Gauss white noise is about 0.01 under the condition of the weak signal to be detected with that group of parameters. and the signal to noise ratio of the system can be obtained by $10\lg[0.5\times(10^{-4})^2/10^{-2}]\approx 63.01\text{dB}$ .This section has successfully realized the detection of weak signal in the non-detection zone through experiment and simulation, proved the validity and feasibility of the proposed removal method of Non detection zone.

V. CONCLUSION

This paper mainly carried out the following two works:

1.Firstly, the equation of improved fractional order Duffing system is derived and the system is constructed. Through the Simulink platform, the model building and function simulation of the improved fractional order Duffing system are carried out. And by comparing with the traditional integer order system, it can be concluded that the fractional order system has obvious advantages in the detection accuracy.

2.In view of the problem of Non detection zone in the same frequency weak signal detection of the above system, a method to remove the Non detection zone is presented, and the elimination algorithm based on the Lyapunov index is proposed. In the simulation experiment, the phase of the weak signal located in the Non detection zone is shifted. and system state is judged accurately with the phase diagram, Lyapunov index, and Poincare interface diagram. the method mentioned in this paper can effectively realize the detection of the signal located in the Non detection zone, and achieved good signal-to-noise ratio and detection anti noise performance.

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